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## ABSTRACT

This document contains the following papers on mathematics from the SITE (Society for Information Technology & Teacher Education) 2001 conference: "Secondary Mathematics Methods Course with Technology Units: Encouraging Pre-Service Teachers To Use Technology" (Rajee Amarasinghe); "Competency Exams in College Mathematics" (Kathy R. Autrey and Leigh Ann Myers); "Computer Aided Personality Assessment of Mathematics Teachers" (Pamela T. Barber-Freeman and others); "AnimalWatch: An Intelligent Computer Tutor for Elementary Mathematics" (Carole R. Beal and others); "Prospective K-6 Educators Attitudes about Technology" (Brian Beaudrie); "Using Databases in Teaching Advanced Mathematics Courses" (Mikhail Bouniaev); "Using Excel To Explore a Thematic Mathematics Unit: Ideas" (Dolores Brzycki and Judi Hechtman); "Mathematics Teachers on Track with Technology" (Laurie Cavey and Tiffany Barnes); "Student Satisfaction with Online Math Courses and Its Impact on Enrollments" (Faith Chao and James Davis); "Teaching Mathematics by Means of 'MathTrainer'" (Fernando Diez and Roberto Moriyon); "Beliefs, Experiences, and Reflections that Affect the Development of Techno-Mathematical Knowledge" (Hollylynn Stohl Drier); "Making Geometry on a Virtual Environment: A Proposal of Continuous Distance Education for Teachers" (Marcia Campos and others); "Helping Elementary Education Majors Brush up on Mathematical Modeling: Insights from a Field Test of a New Online Learning Prototype" (Neal Grandgenett and others); "The Modern Classroom: Using Portable Wireless Computer Networking in the Classroom" (Virgil Varvel and Delwyn Harnisch); "Integrating Mathematics Education Technologies into Teacher Education at the University of Colorado-Boulder" (Jeffrey Hovermill and Michael Meloth); "Teaching Technology with Technology: How One Faculty Member Is Integrating Technology into His Pre-Service Mathematics Methods Classes" (Ken Jensen); "Museums Meeting Schools: Online and Right on the Mark" (James S. Lenze); "Bridging Transformational Geometry and Matrix Algebra with a Spreadsheet-Based Tool Kit" (Anderson Norton and Sergei Abramovich); "Examining Pedagogical Trends within a Graphing Calculator Environment: An Analysis of Pre-Service Teacher Perceptions" (Elliott Ostler and Neal Grandgenett); "System Development and Fundamental Design of Interactive Mathematical CAI System 'MEIKAI'" (Masanori K. Shibata); "Making

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Sense of Number: A Resource for Pre-Service and In-Service Education" (David K. Thomson and Tony van der Kuyl); "Computer Assisted Mathematics Learning Environment - A Study on the Computer, Math and Human Interaction" (Yu-Mei Wang and others); "Numeracy CD - Whole Number Concepts & Operations. Numeracy II CD - Understanding, Using, & Applying Fractions" (Audrey Zelenski); "Technology and Basic Math Skills" (Yuehua Zhang and Michael Patzer); and "An On-Line Math Problem Solving Program the Stimulates Mathematical Thinking" (Joanne Y. Zhang and Leping Liu). Most papers contain references. (MES)

# M A T H E M A T I C S

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It is always exciting to examine each year's mathematics section papers as they come in for the SITE Annual and this year proved to be no exception. Not only do we see a broader representation of mathematical themes and teaching approaches, but we also see an increasing number of international papers. This points to the growing influence of SITE within the national and international community and bodes well for the future of our organization. When we add to this picture the large number of PT3 presentations it is clear to see that SITE is rapidly becoming *the place to be* for technology in teacher education and has made great strides in fulfilling some of the vision of the original founders.

We are also beginning to see some fruits resulting from the maturation of technologies used within the field as a whole. This growing maturity, together with the stabilizing influence of Web based applications, has gone far to providing a more consistent platform within which to explore mathematics in the classroom and teacher education. To a very real extent it is at last possible to seriously examine the impact of technology upon teaching and learning within mathematics without spending two-thirds of the paper dealing with machine specific idiosyncrasies. It's quite exciting to think about what the years ahead might bring when we are able to focus upon substantive issues without having to spend huge efforts in maintaining a stable workplace within which to perform our research.

We thoroughly anticipate that such papers from the future will look markedly different from those of the last 12 years. As we try to imagine what these future papers might look like, it's possible to anticipate changes in at least three areas. The first area would concern itself with the nature of the content to be taught. Mathematics in a technology-enhanced world looks substantially different than it does when the sole tools to think lived are calculator, paper and pencil. Clearly, content issues will need to be addressed in a markedly different fashion in teacher preparation. Secondly, the role of teacher and the interaction between teacher, technology and student will need to be carefully addressed. One might expect to see a growing number of papers dealing with these interactions. The third area of potential change lies in that of the student as they use this new technology enhanced environment to explore mathematics.

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## The Papers

Putting the crystal ball away for another year, we decided to use these three frameworks -*Content*, *Teacher*, and *Student* - to organize and discuss this year's papers. So let us begin by taking a closer look at this sorting rubric. As we organized the papers around these three major themes, we recognized the false distinctions that occasionally arose. For example, one cannot teach without having content or students. However, in nearly every case we are able to reach a consensus regarding which category might serve to best represent the intent of each paper.

Papers that were classified in the *Content* area tended to provide meaningful insight into what the actual mathematics in a technology enhanced environment might look like. And what a fascinating view this set of papers provided! Running the gamut from pre-numeracy through calculus and beyond they provide a fascinating series of insights into the future status of mathematics, as well as the tools with which we will enable our teacher candidates to come to understand and teach it.

Papers that were placed in the second major organization area, *Teacher*, tended to deal more with issues of pedagogy, teacher preparation, and professional development. We also included in this section does papers dealing with the inner life of the teacher and descriptions of general technology tools which might be used across a broad variety of contents and with a broad range of students.

The final theme, students, in many ways is the most important of all. After all, the student is the final consumer in the educational enterprise and should be the center of much of our thinking and planning. As we

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examine the role of student as it emerges from the papers presented this year we see a near requirement for more active, critical, and interactive participation on the part of the student. It is interesting to note that the type of student involvement pointed to in these papers nearly mirrors those outlined by the National Council of Teacher's of Mathematics (NCTM). Relying on the data provided in these papers one can easily build a case that technology can not only be aligned with the NCTM goals, but can easily become a central tool to enable their achievement.

**Content.** Perhaps appropriately, we begin our discussion of the papers in the content section with Autrey and Myers' paper describing their experiences with designing and using competency exams in college mathematics. With today's national trends placing an increasing burden upon teachers to possess high levels of content mastery, such exams are likely to play an increasingly important role in the future. In this paper we see a report on their progress thus far and suggestions for competency exams across the mathematics curriculum. This paper serves to remind us that pedagogical content knowledge, as important as it is, must be thoroughly supplemented with content knowledge of the teacher is to teach with power and authority.

Of course, assessment is of limited value unless we use the results of that assessment to improve student understanding. In Grandgenett, Zygielbaum, Ostler, Hamersky, and Pawloski's paper we see some preliminary findings regarding an online learning program developed in cooperation with the National Science Foundation. Designed to address the specific content requirements of elementary education majors, this program utilizes a unique database structure and should be examined carefully by those planning to implement a similar approach. We look forward to seeing in future presentations the end result of this work.

In examining those papers dealing with a more traditional view of mathematics content, we find numerous technology-enhanced approaches ranging from beginning mathematical concepts to highly advanced mathematics. Beginning with basic numeracy we have Zelenski's discussion of a new program dealing with whole number concepts as well as fractions. It would be well worth examining the *pdf* versions of the annual to see the colorful illustrations that were presented with this paper. This program places mathematics learning within a jungle exploration context and should provide both incentive and interest for the young child.

Continuing in the tradition of placing learning within the context of personally interesting and motivating experiences, Beal, Woolf, Beck and Arroyo's paper describing to *AnimalWatch: An intelligent computer tutor for elementary school mathematics* promises to provide a wonderful existence proof of this effective instructional

approach. It should be noted that thanks to the support of the National Science Foundation *AnimalWatch* will be available at SITE 2001 on a CD-ROM for teacher educators and teachers. We plan on picking up a copy and would recommend that you do also.

Reporting on a project, which includes both CD-ROM and Web based resources; Thomson and van der Kuyl describe the development of their multimedia combined with the Web and CD package geared specifically for the needs of students in the early years of elementary school. This paper truly blended our three categories, as care was taken not only to ensure that the developmental needs of the students were adequately addressed, but also that the materials themselves enabled effective pedagogy on the part of the teachers. The decision to place this paper within the content area was made based upon the perceived focus upon the actual curriculum that was created.

Continuing in this transition from stand-alone CD-ROM to more Web based activities, Lenze's paper provides a fascinating description of some of the design decisions made by museums to meet the interactive learning needs of their Web based patrons. In addition to his careful and insightful commentary on the design process, we found the resources listed within this paper to be extremely worthwhile as they provided many high quality sites in an easy to locate fashion. Although this paper is not listed in the abstracts, we feel that it is definitely worth the time it takes to find it and would hope that his session will be well attended.

The next presentation suffers from the exact opposite problem. Although it is listed in the abstracts it is not present as a paper. This is to be expected given the nature of the interactive session that Miller proposed. Within this session attendees will have the opportunity to examine two inexpensive software products and use them to script out a series of materials designed to teach linear equations. This certainly set sounds like an interesting session and we likewise would hope that it to be well attended.

An absolutely fascinating vision of how transformational geometry and matrix algebra might be taught in a technology enhanced setting is offered by Norton and Abramovich. This paper is highly significant on a number of points. First, the software utilized is off-the-shelf - by this we mean a spreadsheet, in this case Microsoft Excel - was used to generate the workspace within which the mathematics is generated. Secondly, the mathematics itself is quite interesting and would certainly catch student interest. And not least among these considerations the nature of the workspace itself lends itself toward student exploration. The authors have kindly included the Web address for the Excel file comprising the tool kit. We feel that this is a *must* download for teacher educators regardless of the level of their students. A careful exploration of the approach taken by Norton and Abramovich would be

beneficial for anyone interested in using Microsoft Excel, or indeed any spreadsheet, to create such an advanced learning environment.

Finally, Bouniaev provides an insightful discussion regarding the potential for use of databases in teaching calculus and other advanced courses. There is much in this paper of interest not only to the mathematics educator, but the educational theorist as well. It has been fascinating to observe over the past several years the careful theoretical grounding and progress he has made in expanding SSDMA Theory and its applications - not only in terms of information technology, but also within mathematics education.

**Teacher.** At the risk of contributing to the proliferation of yet another buzzword we have chosen to begin our discussion of this section on teaching with Drier's excellent paper on factors affecting the development of Techno-Mathematical Knowledge. Few would argue that the teacher candidate comes into the methods courses equipped with many beliefs and misconceptions regarding content, their role in teaching the content, and the role which technology might play in this effort. In this paper we see a careful examination of shifts in belief occurring during a technology intensive mathematics methods course (additional details concerning this course are available via the Web). The included vignettes illustrate how technology can enable subtle shifts in thinking and attitude regarding not only the use of technology, but also the teaching of mathematics.

Continuing this investigation into teacher beliefs, we have Beaudrie's discussion of perspective K-6 educators attitudes towards technology. Although not listed in the abstracts, this presentation promises to say much about the K-6 teacher attitudes toward technology. As the author correctly notes, the use of technology should not began high school classroom. Yet, in examining the beliefs and attitudes of many elementary classroom teachers we find that many of them come to the technology front sadly lacking in desire to use technology. Referring to the author once more, he presents a poignant observation that although students agree the technology is important to say that they like computers would be an overstatement.

Also on the pre-service front, Amarasinghe's paper describes a technology infusion effort that made use of a set of highly structured technology units in the secondary mathematics course. It is worth reading this paper to see the nature of the design that went behind their creation. We would encourage interested readers to make a point to attend this session.

Moving from pre-service to in-service teachers, in Barber-Freeman, Snead-Greene and McFrazier's paper on computer-aided personality assessment we are shown some of the personality attributes of secondary and middle school mathematics teachers. Furthermore, we are provided tantalizing evidence which indicates that teacher

self awareness of their own teaching and learning styles can prove to be an effective tool for their use in improving instruction.

Making a slight shift in focus, we now examine a series of papers examining various aspects of the teacher preparation process. The first paper in this series from Cavey and Barnes describes a program, *Girls On Track*, requiring active participate part of the student. This summer-based program provided not only opportunities for student growth but also a tremendous experiential base for use by in-service and pre-service teachers. By simultaneously providing a built-in support system as well as immediately relevant and meaningful tasks, this program truly utilizes a variety of approaches towards training teachers. It is an ambitious program, and one well worth watching in the years ahead.

Of course, it is difficult for pre-service teachers to gain appreciation for the role of technology in instruction when the methods professor themselves are unsure of technology's role. In Jensen's paper we are promised some insight into the mind of the methods professor's reflections upon how technology should be integrated into the curriculum. Those people who are still on the fence regarding technology use in the mathematics methods course might wish to attend this session at SITE where this paper is discussed for further information.

Some potential technology roles mathematics teacher education are offered in Hovermill and Meloth's careful analysis of how information technology has been integrated into the teacher education program at the University of Colorado-Boulder. In this paper we see a careful reflections upon the NCTM's goals, together with a very practical series of recommendations into reforming the teacher education program. Their student findings are likewise of interest. First, the students agree the technology offers unique opportunities and enables connections within mathematics and other disciplines. They also came to realize the technology-enhanced investigations lend themselves to authentic collaboration. These are clearly laudable goals in the authors should be well pleased with their attainment.

We would now like to direct your attention away from the development and support of teachers per se and toward some rather unique tools and resources for teacher use. As we began its helpful to remember that not everything works the first time it's plugged in. Quite often in generating new technologies there is substantial time spent thrashing about while we wait for either a stable platform or the correct authoring tools to bring our projects forward. The first paper in this section, by Shibata, offers a case in point. Here we see the beginnings of a description of the creation of not only a dynamic curriculum control module but also a full-blown algebraic editor and authoring system. As is often the case in program development there

are significant startup costs and unforeseen problem are common. Those of you planning entering a substantial phase of program creation might do well to attend this session to listen and learn from their experience.

One solution to the dilemma of developing a stand-alone series of applications lies in utilizing the interactive potential of web-based resources. Although not appropriate for every task, it is amazing how often we can accomplish with Java and HTML what is next to impossible to accomplish with C++. We see this approach utilize to great benefit by Zhang who outlines a set of web-based resources designed to teach fractions. Based upon a very careful semantic analysis of the word actions forming the underlying basis for fraction operations over 100 problems were developed and divided into four basic operations and 11 categories. Although not listed in the paper we would love to see the URL for this particular program as it appears to offer great potential forced widespread teacher reference.

Another interesting paper not indexed in the abstracts is Campos, Gomes, and Neto's work using Tele-ambiente in Brazil. TELE itself is a learning environment composed of the TELE interface and a group of activities utilized by this tool. Based in part upon Microsoft's NetMeeting product this interface has been modified to support mathematics. Although still in its beginning stages, this program offers great potential in providing a meaningful and worthwhile environment for student construction of knowledge as well as enabling teachers to have some unique resources available to them.

Another "not in the abstracts" paper by Diez and Moriyón describes another tool designed for teaching mathematics, that of MathTrainer. Drying heavily upon the programming by example methods math trainer appears to be a powerful visualization tool. Of particular interest lies in the importance of the step-by-step actions being delineated by the teacher. This fits in well the SSDMA notions presented by Bouniaev in this section together with other action on objects models (See Connell, 2000). We look forward to seeing the second-generation of this potentially powerful teaching tool and resource.

In the last in this series of interesting papers that are "not in the abstracts" Varvel and Hamisch describe how wireless computer networks are revamping not only what we think of as a network, but also what we think of as a classroom environment. In addition to describing the logistics is setting up such a wireless network, we also see a well thought out and articulated model describing how such a resource might be utilized. Based upon the student responses to the survey conducted for this paper this technology appears to have arrived just in time to meet some very definite student needs.

Finally from the teacher's perspective we have the opportunity of attending a roundtable session by Williams,

where we will be able to gain insight into one teacher's efforts on integrating technology into math education. This looks to be a very lively session and I would encourage those of you with an elementary interest to attend.

**Student.** Given the increasing reliance upon Web based resources in mathematics teaching, it is clearly of utmost importance to consider our students perceptions about such methods of instruction. In the paper presented by Chao and Davis we are offered a careful look at the impact of putting a large number of mathematics courses online at Golden Gate University. The convenience and flexibility of such online delivery appear to have more than offset the disadvantages of not having face-to-face interaction. It should be noted, however, that these students are predominantly composed of part-time students working full-time. At least for this population the online delivery of mathematics courses make sense.

As has often been suggested, spreadsheets can provide a wonderful environment for the exploration and generation of mathematical concepts. In Brzycki and Hechtman's paper we see how a fifth-grade class was able to successfully use both Excel and PowerPoint to create a very dynamic classroom environment. The authors report that these students not only learned basic financial and mathematical concepts but gained confidence in both creating in sharing their own discoveries.

Recognizing that research, in order to be valuable, must have impact for the population at hand Wang, Swanson and Lam have proposed a replication of some foundational CAI research for the island of Guam. This study as outlined has great potential and we look forward to seeing it's successful conclusion and report. Perhaps a day to the preliminary nature of this proposal, it is not currently listed in the abstracts. Hopefully some preliminary data will be made available during the session itself.

In Zhang and Patzer's research on technology and basic math skills over 104 students in 6<sup>th</sup> 7<sup>th</sup> and 8<sup>th</sup> grades participated in a remedial math lab held over a fourteen-week period. An interesting note to this research was that although it was difficult in the eyes of the authors to see differences between the two groups in terms of math improvement, there were obvious attitudinal changes among the students who had access to the computers. The students reported that they enjoyed working on the computers with their problem solving strategies. An equally important lesson was that simply putting a student in front of a computer does not necessarily mean integration of technology into math curriculum. This is a message which needs to be sent quite often even to those of us who should know better.

On occasion at SITE we often lose track of the fact that for the case of mathematics education the appropriate technology is often the handheld graphing calculator. In Ostler and Grandgenett's study we see how a short

brainstorming session was able to encourage pre-service math teachers to adopt graphing calculator usage. The session for this paper promises to have insight both in terms of calculator adoption as well as field based aspects of teacher education. We encourage those of you with interest in these areas to attend this session.

### **Concluding Remarks**

Mathematics and Mathematics Education have each had long and distinguished histories, which like many other aspects of our world are becoming deeply and profoundly impacted by technology. The papers in this year's section serve to remind us that whatever the organizing screen we use - *Content, Teacher, or Student* - or that of our own devising, it is impossible to use old labels to predict the new and emerging trends. As a group, these papers serve to remind us not only of the changes which technology has brought about in foundational issues of mathematical content, but also that our interactions with one another, with our students, and with this changed content will forever be enhanced by these new tools of technology.

We hope that the stability of the modern technology platform will enable us to concentrate more on researching questions of substance rather than on maintaining a metastable platform in the midst of logistical and hardware difficulties. Research of this type will prove invaluable in advancing our field of understanding so we might take better advantage not only of the new tools we have to think with, but also the new understandings which they might enable.

### **References**

Connell, M. L. (2000). Actions on Objects: A theoretical framework for mathematics education. In Willis, D., Price, J. D., & Willis J. (eds.) Technology and Teacher Education Annual 2000. (pp. 1034-1039). Charlottesville, VA: Association for the Advancement of Computing in Education.

## Secondary Mathematics Methods Course With Technology Units: Encouraging Pre-service Teachers to Use Technology

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**ABSTRACT:** Although technological tools that can be used in the mathematics classroom were introduced to prospective teachers at various stages in their undergraduate curriculum, novice teachers seldom use them in their actual teaching. This paper describes the efforts taken to encourage secondary mathematics teachers to use technology in their teaching by introducing five highly structured technology units into the secondary methods class. The five units are designed to use Internet, Graphing Calculators, Geometer's Sketchpad, Computer Based Laboratories (CBL), and Spreadsheets. The intent of these units is to make students aware of the existing resources, provide hands on experience, look at the resources critically, and collectively develop effective ways to integrate these resources to mathematics teaching.

### Background

*Methods and Materials in Secondary Teaching* is a required upper division course for prospective secondary school mathematics teachers at California State University, Fresno. Students are expected to learn various teaching methods and materials/resources that help them to become effective mathematics teachers in this class. Although technological tools that can be used in the mathematics classroom were introduced to them at various stages in their undergraduate curriculum, students seldom use them in their actual teaching. Various researchers have tried to identify the reasons for their reluctance to use available technological tools in their classrooms (Flake, 1990; Fine & Fleener, 1994; Brooks, D., & Kopp, T.W., 1990). The research findings promote the change of students' beliefs, hands-on experience, group/collaborative work, reflective thinking, and oral communication in mathematics education classrooms. Studies revealed that just learning about technology is not enough to unlock the minds of the preservice teachers to the possibility of those resources as instructional tools. For example, Fine & Fleener, (1994) concluded their study saying "the mention of pros and cons of calculator use and the brief demonstration lesson plan given in the math methods courses left these preservice teachers with at best a superficial understanding of this form of technology. However they appear inadequately prepared to employ calculators in classrooms of their own."

In order to encourage secondary mathematics teachers to use technology in their teaching by infusing technology units into the secondary mathematics methods course, five technology units were introduced to the secondary methods class. The five units are 1) Internet; 2) Graphing calculators; 3) Geometer's Sketchpad; 4) Computer Based Laboratories (CBL); and 5) spreadsheets.

### Nature of the Efforts Taken

The intent of these units is to make students aware of the existing resources, provide hands on experience, look at the resources critically, and collectively develop ways to integrate these resources to mathematics teaching effectively. To accomplish these goals, each unit was designed with six components: 1) demonstration; 2) classroom discussion of the resource; 3) students' group projects; 4) student-designed mathematics lessons; 5) sample teaching of the lesson; and 6) peer evaluation and critique of the lessons.

The demonstration is designed to introduce the components of the resource to students. In this part students were given a brief introduction followed by a sample use of the resource in teaching a lesson in mathematics. The classroom discussion is designed to discuss the pros and cons of using the resource in a



mathematics classroom by reviewing various research studies and position papers. The goal was to challenge students' existing beliefs about the use of the resource. Researchers suggested that unless students' beliefs about the usefulness of the technology is changed they are not going to use these in their own teaching (Flake, 1990). The projects are designed to familiarize students with the resources via group projects where students inquire about these resources in collaborative groups. The student-developed lesson plans and sample teaching units are the major portion of this effort. In these two parts students are supposed to incorporate the resource they are learning to enhance a mathematical topic they are interested in teaching. For example, students used Geometer's Sketchpad to enhance a lesson on graphing linear equations or used graphing calculators to enhance a lesson on teaching quadratic equations. The last part, the evaluation and critique, is a classroom discussion designed to promote reflection on students' own teaching experiences, oral communication and discussion of teaching methodologies. During these discussions students are expected to receive a constructive feedback from the classmates and explain the reasons behind selecting the mathematical topic, and the activities incorporated into their lesson.

The ongoing study will include details of the student experience in the class, and the results of the survey at the end of the semester will be used to measure the effectiveness of the units. Students' feedback related to the usefulness of the units in their actual student teaching experiences will be also collected along with student-developed sample lesson plans.

## References

- Fine, A.E., & Fleener, M.L. (1994). Calculators as a instructional tools: Perceptions of three preservice teachers. *Journal of Computers in Mathematics and Science Teaching*, 13(1), 83-100.
- Flake, J.L. (1990). Preparing teachers to integrate computers into mathematics Instruction. *Journal of Computers in Mathematics and Science Teaching*, 9(4), 9-16.
- Brooks, D., & Kopp, T.W. (1990). Technology and teacher education. The subject matter preparation of teachers. In W.R. Houston (Ed.), *Handbook of Research on Teacher Education*. (pp. 498-513). New York: MacMillan Publishing.

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## Competency Exams in College Mathematics

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**Abstract:** The idea of competency exams as an integral part of a college mathematics course started at Northwestern State University because of the move to reform based calculus. Further changes in our curriculum have led to a more widespread use of competency exams. While implementation of these exams has been successful in our calculus courses and a course for elementary education majors, we wish to expand our competency exams into developmental mathematics and College Algebra. These exams will ensure that our students have basic analytical skills, while allowing us to maintain reform based courses. We hope to use computer and internet based exams in order to manage a competency exam program throughout the department. We are currently testing a computer based, self-paced program in one developmental math class. We will report on the process we use for these exams and the results we have seen from competency exams across the mathematics curriculum

### Our History

The idea of competency exams as an integral part of a college mathematics course started at Northwestern State University because of the move to reform based calculus. Further changes in our curriculum have led to a more widespread use of competency exams. As a part of the reform based calculus, we place a greater emphasis on numerical and graphical ideas, with less emphasis on the analytical skills that previously dominated the courses. While we believe that a reform based approach leads to a greater understanding of the concepts studied in mathematics, our students were naturally less experienced (and less competent) with algebraic manipulations and other analytical skills. In order to ensure that our students have the ability to perform basic analytical operations and the ability to teach the analytical processes, we have implemented competency exams that students must pass as a requirement of the courses.

In recent years, we have developed two new mathematics courses specifically for elementary education majors. The first of these courses, designed to be taken after a student completes College Algebra and Finite Mathematics, focuses on the concepts and applications of mathematics needed in elementary and middle school grades. We were surprised to notice a deficiency in the basic mathematical competence of students in this course (soon to be elementary school teachers). In order to address this problem, we developed a competency exam as a requirement for this course as well. The exam focuses on problems involving computations with fractions, percents, and basic equations. Students are not allowed to pass this course and become teachers without passing the competency exam.

Having noticed this deficiency within the elementary education majors, we believed that the same would be true for other students. Furthermore, we felt that the deficiencies needed to be addressed at an early stage in the students' preparation. At that point, the decision was made to consider the introduction of competency testing in our college algebra course as well as the developmental math courses. Development of

fluency in mathematics requires students to be proficient with computations as well as concepts. As stressed in NCTM standards, understanding without the necessary computational proficiency inhibits the ability to become efficient problem solvers; the competency tests we are currently using and those we plan to use emphasize that computational ability.

Over half of the students entering the university are in need of some form of remediation; generally, the majority require remediation in mathematics. At Northwestern, students are placed into college algebra on the basis of either their ACT math scores or their scores on the math placement exam. About 900 students enroll in developmental math courses offered on the main campus in the fall semester and about 600 enroll in the spring semester. Many of these students lack basic arithmetic skills as well as elementary algebra concepts. They not only lack computational skills but also the conceptual understanding required for efficient problem solving.

In an effort to better utilize existing resources, the decision was made to offer the beginning developmental course as a self-paced course rather than the traditional lecture course. A pilot section was offered in the fall of 2000 using the existing technology for our online college algebra course to create a hybrid type course. Students still attend class but work in an online classroom setting. This allowed faculty to incorporate competency testing as a component of the course requirements.

## **Implementation**

The competency exams given in calculus courses have been a successful addition to the requirements of the courses. The exams are composed of about 15 problems involving basic differentiation and integration techniques. Students may retake the exams as needed to achieve a minimum competency standard (currently 85%). Since we are a small department, offering only one section of each calculus course per year, the instructor easily manages these tests and re-tests. There are typically 15-25 students in each course. On average, each student requires two attempts to pass the test. Students can schedule times to take the tests, and instructors can find time to grade them within normal office hours.

The competency exam in the course for elementary education majors is a valuable tool to ensure that our students do not graduate without the mathematical abilities they will need in the classroom. Many of the students have been successful in college mathematics, even without a solid background in some of the mathematical concepts that are typically overlooked at this level. There are two sections of the course each semester, with a total of about 60 students. These students require about five attempts to pass the test, making the exams time consuming for the students as well as the instructor, since the tests are scheduled outside the normal class time. Students are not allowed to pass this course and become teachers without passing the competency exam.

Topics covered in the developmental mathematics course include: arithmetic operations with whole numbers, fractions, and decimals; problems involving ratios, proportions, and percents; fundamental operations with sign numbers. To ensure students have obtained at least minimum competency in these areas before moving to the next level of developmental math, four competency tests were created. Students must pass each of these with a minimum score of 80%; failure to do so means that the student will fail the course. Using the technology available through Blackboard (the platform we are currently using for our online course offerings), students may take the tests as many times as necessary in order to achieve the minimum competency. Each test is randomly generated from a test pool and is password protected. Test proctors enter the appropriate password for each test that the student wishes to take. Preliminary results indicate students require, on average, 3 attempts per test to achieve at least the minimum level of competency and those who have passed the competency exams are performing at higher levels on regular exams given during the semester than those who have not passed the tests.

## **The Future**

The college algebra course (Math 1020) taught at Northwestern is also a reform-based course. Since nearly every student takes this course, competency testing is an important element. In talking with faculty members across campus (College of Education, College of Business, Industrial Technology, Biology, etc.), the

most common complaint was the lack of basic algebraic skills by many of the students who have completed their math sequence. What we have seen repeatedly is some understanding of basic concepts involved in solving a problem but a lack of fundamental algebraic and other analytical skills necessary for reaching a solution. In an effort to address this deficiency, faculty at Northwestern have proposed the addition of basic competency testing as an additional course requirement. By doing so, we believe that those students entering the teaching profession will be better prepared to teach mathematics at all levels.

The main issue currently under consideration is the implementation of such testing. Our college algebra classes generally have enrollments of 50 to 60 students per section (about 20 sections are offered each semester). Paper and pencil testing with unlimited retakes would make the task almost totally unmanageable. Use of technology, such as that currently being used in the developmental class, would provide not only a more manageable means of test administration but also a method for generating multiple versions of the test. Use of passwords for the test would address security concerns; however, there are still issues to be resolved in this area.

## **Conclusions**

Overall, faculty believe that the use of competency testing as an additional course requirement will provide some measure of assurance that students have the basic mathematical skills needed for success in the future. For many of the students, their level of motivation and self-efficacy has improved with the addition of these competency tests. They are now able to see the link between conceptual material and computational proficiency. In the developmental math class in particular, students have commented that the tests forced them to study the material and finally master it. We believe that the addition of these tests to the college algebra course will have a similar effect.

# COMPUTER AIDED PERSONALITY ASSESSMENT OF MATHEMATICS TEACHERS

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## ABSTRACT

Mathematics teachers from the state of Mississippi were invited to participate in a training seminar, Technology, Reading, Riting, Rithmetic ( $TR^3$ ), during the month of July 2000. The overall purpose of  $TR^3$  was to provide mathematics teachers with effective teaching strategies for improving achievement of Mississippi students. In addition, teachers were provided intensive training with a "hands-on" curriculum, based on the belief that the learner must be an active participant in the learning process and must construct his/her own ideas through reflective thought. This objective was met through the utilization of personality assessments. Research suggests teachers realize that optimal learning is impacted by understanding personality preference (Alcock, 1998). Thus, the purpose of this research was to examine personality test that assessed personality, teaching and learning styles of middle school and secondary mathematics teachers.

## Background

Mathematics has been identified as the critical filter for careers in science, technology, engineering and mathematics; thus, it is crucial that all students gain proficiency in mathematics so they can successfully participate in these careers. According to a National Science Foundation (NSF, 1993) report, "Shaping the Future," too many students do not enroll in mathematics courses because they find them "dull and unwelcoming." This report also found teachers lacking the excitement of discovery and the confidence and ability to help students engage in mathematical knowledge. Thus, as part of the overall educational reform initiative, there must be a concerted effort to improve mathematics education in the United States by emphasizing various mathematical concepts that have been educationally, financially and politically neglected. Also, teachers of mathematics must find innovative and effective ways for encouraging students to have an appreciation for and to achieve at higher levels in mathematics. With these overarching themes in mind,  $TR^3$  will be a tool for improving professional skills and providing secondary mathematics teachers the opportunity to acquire knowledge needed to empower all students in the areas of mathematics and technology education.

$TR^3$  was designed to improve the skills of mathematics teachers of grades 7-12 through intensive professional development in methods empirically proven to promote and sustain high student achievement in mathematics. The project objectives are to 1) strengthen teachers instruction in seventh and eighth grade math, pre-algebra, algebra I & II, geometry, probability and statistics, 2) provide technology training for teaching mathematics in grades 7-12; 3) provide strategies for integrating language arts (reading and writing) instruction into the

teaching of mathematics, 4) address the equity/ diversity/multicultural perspectives necessary for teaching math in a diverse classroom and 5) demonstrate how to assess the impact of teachers' instruction on students. Research has shown that a large number of teachers need further training in content areas that have been identified by the NCTM Standards. Thus, *TR*<sup>3</sup> focused on increasing teachers' awareness, and also provided training and application of mathematics in specific content areas. In many cases, content areas were not included in particular courses taken by the in-service teachers. Therefore, it is necessary to bridge the gap between the prior training of in-service teachers and the expectations of the NCTM Standards (NCTM 1999). Teachers were provided intensive training with a hands-on curriculum, based on the belief that the learner must take an active role in the learning process and must construct his/her own ideas through reflective thought. The curriculum for the mathematical institute was guided by the National Council of Teachers of Mathematics Curriculum and Evaluation Standards and the Mississippi Curriculum Structure for Mathematics. In addition, in service teachers were introduced to new mathematical concepts and content, current strategies in teaching the diverse learner, as well as methods of teaching and assessment. All topics were built on the *Questioning Approach* to teaching. Throughout the project, activities performed were hands-on (manipulatives), based on inquiry, stressing critical thinking skills, and utilizing cooperative learning strategies. The project goals were chosen because of their direct impact on the learning of students.

### **Technology**

Just as students should be provided a problem-solving environment (NCTM, 1989), it is essential to provide in-service teachers with a problem-solving training environment for learning how to identify, explore, and solve problems using manipulatives and technology. It is equally important to be aware of how the integration of technology can affect students' opportunities to learn important mathematics concepts.

Technology can effectively transform teaching and learning by:

- a) Moving the focus of instruction from whole class to small groups; changing the primary mode of instruction from lecture and recitation to coaching;
- b) Requiring students to be explorers, investigators, thinkers and workers rather than passive recipients of pre-digested information;
- c) Allowing individual learners to pursue areas of interest in depth rather than requiring all of them to learn the same material on the same day in the same way; providing teachers with more time to work with students who are most in need while those who are most capable advance at a pace appropriate for them;
- d) Creating opportunities for assessment based upon products and real tasks rather than solely upon traditional tests;
- e) Promoting cooperation and collaboration rather than competition; and
- f) Making it possible for teachers to address diversity more effectively rather than teaching primarily to the mean. (United States Department of Education, 1996, p. 2)

In addition to electronic technology, participants had opportunities to use other math manipulatives. Sufficient repetition of events through the use of hands-on materials should occur in order for ethnically diverse students and females to obtain an internal locus of control with regard to mathematics (Freeman, 1995). Further support for use of hands-on manipulatives involved cognitive theory. Piaget believed that cognitive growth occurs best when there is repetitive use of concrete materials in the learning process (Piaget and Inhelder, 1975). Consequently, teachers participated in the computer aides personality test rather than the traditional method of assessment.

### **Equity/Diversity/Gender**

Culturally and ethnically diverse populations within public schools are increasing to the point that minorities now comprise the majority population in 25 of the largest city school systems in America. This growing phenomenon suggests that educational strategies must adequately address how to prepare all United States students so they can successfully compete in an ever-growing technologically dependent world. Our science, engineering, technology and mathematics work force will be in jeopardy of not meeting the needs of the nation, unless there is an increase in higher order thinking proficiencies in both mathematics and technology for the culturally and ethnically diverse students at the middle and high school levels. Instruction must be taught to promote participation in and understanding of mathematics for diverse students.

Eighth and tenth grades have been identified as the pivotal points for encouraging females and ethnically diverse students to pursue higher mathematics courses (Freeman, 1995). An additional aspect of the *TR*<sup>3</sup> training ensured that teachers were aware of the varying learning styles and personality preferences which

should help lead to an increase in the number of females and ethnically diverse students enrolling in higher mathematics courses.

In order for in-service teachers, students, and administrators to reach their educational, professional and personal potential, they must be actively involved in developing and implementing instructional models. The primary outcome sought through this research was to promote what Franke, Carpenter, Fennema, Ansell, and Behrend (1998) call "self-sustaining, generative change." Self-sustaining, generative change is when teachers make changes to improve their methods of instruction to ensure continued growth and problem solving. To ensure that self-sustaining, generative change is on-going, a teacher personality assessment was conducted. One's personality is without question the most important driver influencing career choice, relationships, health and sense of well-being (Shaughnessy, 1998). To understand the full potential of one's personality, it is critical to first measure and then gain insight into your strengths and developmental needs through examining the results of personality assessment.

## What is Personality Assessment?

Personality assessment, as a scientific endeavor, seeks to determine those characteristics that constitute important individual differences in personality, to develop accurate measures of such attributes, and to explore fully the consequential meanings of these identified and measured characteristics (Ozer & Reise, 1994). Simply stated, personality assessment measures such traits as motives, intentions, beliefs, and styles (Wiggins & Pincus, 1992). This measurement can be used to gain supplementary insight into an individual's temperament or character (Keirsey, 1998).

There are a multitude of personality type inventories available for use in the educational setting. These days, taking the Myers-Briggs Type Indicator (MBTI) or the Keirsey Character Sorter and using the results to determine the teaching styles of educators, the leadership styles of administrators, or the learning styles of students is not unusual. For all intent and purposes, we chose to utilize the Keirsey Character Sorter to measure specific personality styles of mathematics teachers.

The Keirsey Character Sorter attempts to help individual's understand both their temperament and character through an investigation of personality differences. In his recent book, *Please Understand Me II*, Keirsey (1998) introduces the definition of temperament by asking the following insightful question, "What, we might ask, is this thing called "*temperament*," and what relation does it have to *character* and *personality*?" This is an important question to consider when attempting to measure and define one's personality. Keirsey observes that there are two sides to personality: One is temperament and the other is character. Temperament is a set of inclinations we are born with, while character is a set of habits we acquire as we grow and mature. Character is disposition, developed over a lifetime; temperament is predisposition, hardwired in from birth. And, as temperament plays an important mediating role in efforts to assess and understand differences and to facilitate human development (Tice, 1994), an understanding of how a combination of the two creates one's unique personal style is very important when we seek to examine or assess learning styles and teaching styles as related to personality.

### *Personality Assessment: A Measurement of Learning Styles and of Teaching Styles*

Self-assessment is an in-depth look at who you are. It is the process of discovering and learning your personality type; becoming aware of what's important to you; and understanding yourself (i.e., your values, interests, aptitudes, abilities, strengths and weaknesses). It is also a way to gain knowledge of what is a good fit or match for your personality (Ellis, 1999). As one begins to explore his or her learning style or teaching style, they will first need to understand themselves.

Learning styles have been defined as physiological, cognitive, and affective behaviors that serve as relatively stable indicators of how learners perceive, interact with, and respond to learning environments (Keefe, 1987). It is the way a person concentrates on, processes, internalizes, and remembers new and difficult academic information or skills. Styles often vary with age, achievement level, culture, global versus analytic processing preference, and gender (Shaughnessy, 1998). Likewise, teaching styles have been defined as either progressive or traditional. The *progressive* style is characteristic of the following: (1) integrated subject matter; (2) teacher as guide to educational experiences; (3) active pupil role; (4) pupils participate in curriculum planning; (5) learning predominantly by discovery techniques; (6) intrinsic motivation, where external rewards and punishments are not necessary; (7) not too concerned with conventional academic standards; (8) little testing; (9) accent on co-operative group work; (10) teaching not confined to classroom base; (11) accent on creative expression; while the *traditional* style is characteristic of the following: (1) separate subject matter; (2)

teacher as distributor of knowledge; (3) passive pupil role; (4) pupils have no say in curriculum planning; (5) accent on memory, practice and rote; (6) extrinsic motivation, where external rewards are used; (7) concerned with academic standards; (8) regular testing; (9) accent on competition; (10) teaching confined to classroom base; (11) little emphasis on creative expression (Francis & Grindle, 1998). Although the above information indicates that unlike learning styles, teaching styles may be characterized more so by preference than aptitude, both learning styles and teaching styles are thought to be stable and enduring personal qualities and not easily acquired (Derry & Murphy, 1986; Apple, 1999).

## Methodology

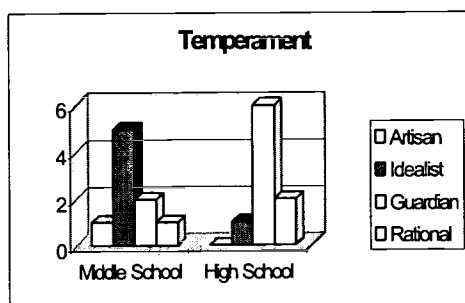
Schools are a wonderful and vast mix of unique teachers and students with individual educational styles and perspectives. Hence, it is imperative that teachers be knowledgeable of the impact that personality style has upon optimal learning. In order for teachers participating in Technology, Reading, Riting, Rithmetic (TR<sup>3</sup>) Seminar, to develop a better understanding of the importance of personality, teaching and learning styles, they were required to complete a personality assessment via Internet. According to Wenzlaff (1998), teachers of today and tomorrow must first know themselves and how those intangibles will guide them in their classroom before that can effectively empower their students.

Research has shown that a large number of teachers need further training in content areas that have been identified by the NCTM Standards. Thus, TR<sup>3</sup> not only increased teachers' awareness, but also provided training and application of mathematics in specific content areas through the integration of technology. Teachers were provided intensive training with a "hands-on" curriculum, based on the belief that the student must take an active role in the learning process and must construct his/her own ideas through reflective thought. In meeting this objective, personality assessment was vital in enlightening teachers as to how their own teaching and learning styles, beliefs and values will impact learning within their classroom. Because teachers should be aware of the impetus of personality style/preference on optimal learning (Alcock, 1998), participants were given the opportunity to complete on-line personality assessments. Following the completion of the survey, counselors assisted with the explanation and interpretation of survey results.

Twenty middle school and high school teachers from the state of Mississippi participated in a Mathematics Summer Institute sponsored by the Institutions of Higher Learning in the state of Mississippi during the summer of 2000. Eighteen of the 20 teachers completed the on-line personality assessment. Nine of the participants were middle school teachers and nine were high school mathematics teachers. *Figure 1* identifies the breakdown of the mathematics teachers' temperament according to grade levels. According to Keirsey (1998), those individuals with an Artisan portrait are concrete in communicating while utilitarian in instituting goals. Where as, the Idealist is an abstract communicator and cooperative in goal implementation while the Guardian is concrete in communicating and cooperating in the implementation of goals. The individual with the Rational portrait is abstract in communication and utilitarian in the implementation of goals, however, they are highly skilled with strategic analysis.

*Figure 1* suggests that of the nine middle school teachers the majority of their portraits were Idealist. According to Keirsey (1998), the Idealist is usually found teaching, counseling, mentoring or tutoring. In addition, Idealist comprises approximately eight to ten percent of the population. The greatest temperament found among the high school teachers was Guardian. The teachers portraits that reflected Guardian are identified as supervisors, inspectors or protectors. Individuals that projected the Rational portrait are planners, inventors or engineers. In *Figure 2*, the character style of the majority of middle school teachers were "Teachers," while the majority of the high school teachers character style was the Protector.

Figure 1





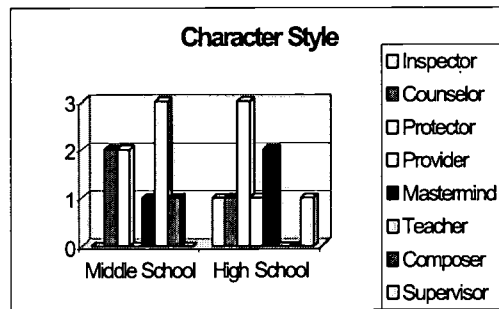


Figure 2

## Conclusion

### The Keirsey Character Sorter as a Personality Assessment Tool

Temperament, which helps us to understand why we process and respond to situations as we do and why others may process and respond differently in terms of the same phenomena, can operationally be defined through four dichotomous traits: 1) extraversion -E and introversion -I, 2) sensing -S, and intuition-N, 3) thinking -T, and feeling -F, and 4) judging -J and perceiving (Myers & Kirby, 1994). Keirsey and Bates (1984) describe four basic temperaments that can be derived from the interaction of these types of traits, each temperament having its own primary or core value. Keirsey (1998) utilizes the previously stated traits to explain temperament, character and personality as either Artisan, Idealist, Guardian or Rational.

Personality factors, teaching philosophy, and preferred learning styles can play an important role in effective teaching and effective learning in the classroom. According to recent research, the blending of personalities and teaching styles is likely to have a great impact on course success, course continuance, and teacher and student satisfaction. A satisfying (or not so satisfying) teaching situation will affect how the teacher relates to the student; that in turn, affects how the student views the success of the academic course their interest in continuing in such a curriculum (Bruneau-Balderrama, 1997). Fairhurst and Fairhurst in their 1996 studies on how temperament and personality type theories can affect teaching and learning call this type of classroom satisfaction the "personality connection."

In conclusion, personality assessment as used during the *TR<sup>3</sup>* Mathematics Summer Institute received favorable responses from the participants. Some responses from the completion included: "Now, I understand why I teach like I do..." "...I feel this will really help me within the classroom..." and several teachers were anxious to have their students to complete personality assessment via e-mail. Finally, this research concurs with Wenzlaff (1998) findings that teachers of today and tomorrow must first know themselves and how those intangibles will guide them in their classroom before that can effectively empower their students.

## References Cited

- Alcock, M.W. (1998). Repechage, reflection, and brain processing: Personality influences in the classroom. *Nassp Bulletin*, 82(598), 56-62.
- Apple, M.W. (1999). Experiencing School Mathematics: Teaching Styles, Sex and Setting. *Educational Policy*, 13(2), 333-336.
- Bruneau-Balderman, O. (1997). Inclusion: Making it work for teachers too. *Clearing House*, 70(6), 328-330.
- Derry, S.J. & Murphy, D.A. (1986). Designing systems that train learning ability: From theory to practice. *Review of Educational Research*, (1), 1-39.
- Ellis, M. (1999). Self-assessment: Discovering yourself and making the best choices for you! *Black Collegian*, 30(1), 30-34.
- Fairhurst, A.M. & Fairhurst, L.L. (1996). *Effective Teaching, Effective Learning: Making the Personality Connection in Your Classroom*. Palo Alto, California: Davies-Black.

- Franke, M. L., Carpenter, T., Fennema, E., Ansell, E., and Behrend, J. (1998). Understanding teachers' self-sustaining, generative change in the context of professional development. *Teaching and Teacher Education*, (14)1, 67-80.
- Francis, L.J. & Grindle, Z. (1998). Whatever happened to progressive education? A Companion of primary school teachers' attitudes in 1982 and 1996. *Educational Studies*, 24(3), 269-279.
- Freeman, P. (1995). Ethnic and gender equity in mathematics via computer technology. *The Mid-South Educational Research Association*.
- Keefe, J.W. (1987). Learning style theory and practice. Reston, Virginia: NASSP.
- Keirse, D. (1998). *Please Understand Me II: Temperament, Character, Intelligence*. Del Mar, California: Prometheus.
- Keirse, D. & Bates, M. (1984). *Please Understand Me: Character and Temperament Types*. Del Mar, California: Prometheus.
- Myers, K. & Kirby, L. (1994). Introduction to Type Dynamics and Development: Exploring the next level of Type. Palo Alto: California: Consulting Psychologists Press.
- Myers, I.B. & McCauley, M.H. (1985). *Manual: A guide to the development and use of the Myers-Briggs Type Indicator*. Palo Alto, California: Consulting Psychologists.
- National Council of Teachers of Mathematics (1999). *Overview of Standards for Grades Pre-K-12*. Reston, Virginia.
- National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia.
- National Council of Teachers of Mathematics (1989b). *Professional Standards for Teaching Mathematics*. Reston, Virginia.
- National Council of Teachers of Mathematics (1995). *Assessment Standards for School Mathematics*. Reston, Virginia.
- Piaget, J. and Inhelder, B. (1975). *The Origin of the Idea of Chance in Children*. New York: Norton.
- Ozer, D. J. & Reise, S.P. (1994). Personality Assessment. *Annual Review of Psychology*, 45, 357-388.
- Shaughnessy, M.F. (1998). "An interview with Rita Dunn about learning styles." *Clearing House* 71(3), 141-145.
- Tice, T.N. (1994). Temperament. *Education Digest*, 59(5), 50-51.
- United States Department of Education (1996). Goal 5: First in the world in math and science technology resources. *Achieving the Goals*.
- Wenzlaff, T.L. (1998). Dispositions and portfolio development: Is there a connection? *Education*, 118(4), 564-572.
- Wiggins, J.S. & Pincus, A.L. (1992). Personality: structure and assessment. *Annual Review of Psychology*, 43, 473-504.

## **AnimalWatch: An intelligent computer tutor for Elementary School Mathematics**

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**Abstract.** AnimalWatch is a computer based tutor for mathematics instruction, designed for use by 10-12 year olds. AnimalWatch integrates mathematics problem solving with adventure narratives about endangered species (Atlantic Right Whale; Giant Panda; Przewalski Wild Horse). Whole number operations, and like and unlike fractions are included. AnimalWatch uses artificial intelligence algorithms to model individual student performance and continually adjusts its problem selection, instruction, and hints so that each student is challenged without becoming overly discouraged. AnimalWatch is available on CD-ROM (both Macintosh & Windows platforms) for teachers and teacher educators and will be distributed at SITE 2001. A web site provides a forum for comments, updates, support, and authoring tools for teachers to create their own mathematics problems and endangered species adventures.

### **Project Goals**

There are many concerns about the poor level of mathematics achievement by students in the United States, as well as the lack of interest in studying mathematics that can be apparent at the transition from elementary to junior high school. This dislike and avoidance of mathematics has been particularly noted among female students, and among members of traditionally underrepresented groups. Much research has sought to identify the factors that contribute to math avoidance, including gender role stereotypes, peer pressure, gender-biased instruction and expectations on the part of parents and teachers, poor preparation for mathematics teaching on the part of classroom teachers, among others. Although the role of teachers may well have been somewhat overemphasized in the popular press in recent years, it is still the case that many new teachers at the elementary level report feeling ill-prepared to teach mathematics and that they prefer to focus on reading. To address these concerns, teacher educators need well designed, easy to use resource materials that can support effective and engaging classroom instruction in math.

One way to address concerns about the quality and effectiveness of math instruction is to use well designed, multimedia materials to enhance learning. The goal of the AnimalWatch project is to use the power of an artificially intelligent, multimedia computer tutor to provide high quality mathematics instruction that will engage both male and female students, increase their math self confidence, and encourage them to continue to study math. With the support of the National Science Foundation, AnimalWatch will be available at SITE 2001 on CD-ROM for teacher educators and teachers to use in their programs and classrooms.

### **Program Description**

AnimalWatch is a computer program that is easily installed via CD-ROM. It will support roughly 10-14 hours of use by students in the 4<sup>th</sup>, 5<sup>th</sup> or 6<sup>th</sup> grades. AnimalWatch runs under both Windows and Macintosh operating systems. The program is self contained, meaning that it can be launched by the student and used without full time, direct teacher supervision or input.

**Mathematics in Context.** When the student starts the AnimalWatch program, he or she first selects an endangered species: Atlantic Right Whale, Giant Panda, or Przewalski Wild Horse (Figure 1). Then, the student begins a narrative in which math problem solving is integrated with information about the chosen species. For each animal, the narrative includes four distinct contexts. For example, in the case of the

Prezwalski Wild Horse (also known as the Takhi Horse), the first context includes problems about the unique characteristics of the Takhi along with background on how it became extinct in the wild. The second context provides information about how the Takhi species was preserved in zoos around the world and on efforts to save the animals. The third context focuses on the history, geography and culture of Mongolia, the original home of the wild Takhi. In the final context, the student plans and raises funds for a trip back to Mongolia with zoo-raised horses that are being released back into the wild at the Hustain Nuruu Nature Preserve.

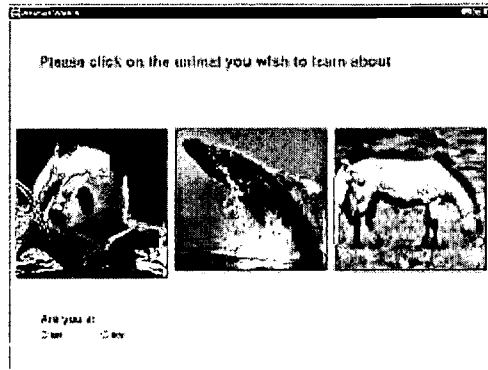


Figure 1: Endangered species selection in AnimalWatch

**Math Problem Template Bank.** Each of the four contexts provide dozens of problem templates that are based on factual information about the endangered species and their habitats (see example in Figure 2). For example, in the case of the Giant Panda, math problems involve research at the library about the Panda and its habitat, reading about the birth of a new Panda in captivity at the San Diego Zoo, the reviewing of maps of the Panda's habitat, estimates of the expenses associated with a trip to China, and analyses of the rate of decline of the Panda population over time, etc. Each math problem includes an image or graphic that is tailored to the problem, e.g., a map of Cape Cod bay showing the migration route of the Right Whale for a problem in which students must calculate the fractional progress of a whale pod over the course of a week's travel.

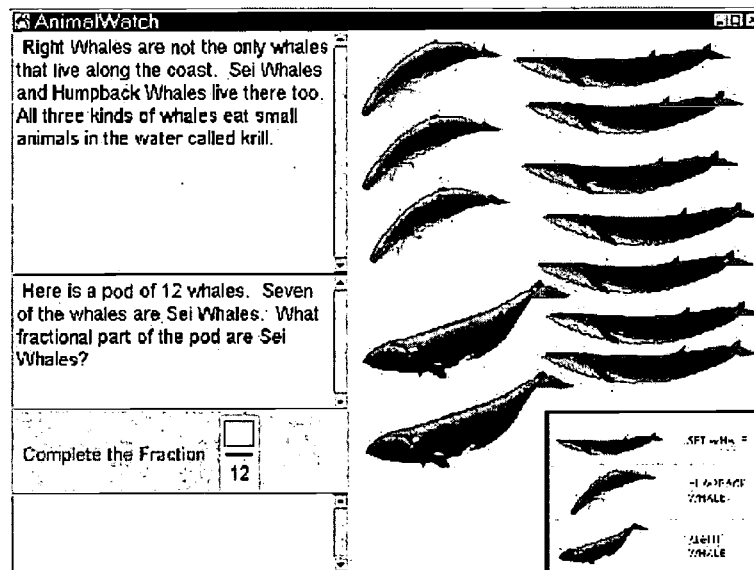


Figure 2: Example of word problem about the Right Whale

To date, the problems in AnimalWatch include whole number operations (addition, subtraction, multiplication, and division of numbers up to 3 digits), introduction to fractions, and addition and subtraction of like and unlike fractions (up to 2 digit denominators). In most cases, the specific math problems are custom created "on the fly" by AnimalWatch. The system uses about 1200 unique problem templates that are instantiated with specific numbers and operations that are tailored for individual users. For each animal and each context, problem templates are available for all mathematical operations included in AnimalWatch. This allows students to work at different levels of math problem solving while also making progress through the narrative at roughly the same pace. (This avoids the potentially demoralizing situation in which one student has finished the story whereas the student at the adjacent computer is still on the first context.) After reaching the end of one narrative, students can also change to a different endangered species and continue working at the same level of mathematics; for example, if a student was working on addition of single digit like fractions at the end of the Panda narrative, he or she would start with like fractions when moving on to the Right Whale narrative. AnimalWatch continually generates new problems from templates, rather than drawing on a fixed, stored set of pre-prepared problems, students who are strong in math can work for longer periods on the most difficult problems, while their less skilled classmates can progress at the easier levels.

**Help, Hints and Instruction.** AnimalWatch has been designed as a supplement to classroom instruction in math, and was developed with master teachers to incorporate the vocabulary and materials (e.g., Cuisenaire rods) that students would encounter in the classroom. Although it is not designed to offer primary instruction, it does include many hints and instructions screens that will guide the student through the problem to the solution (see example in Figure 3). The particular hint or help screen provided is selected by the system based on its estimate of what the student understands and where there might be a misconception. For example, AnimalWatch might reason that a student who has made several errors in adding unlike fractions might need a hint about how to find the least common multiple, whereas a classmate who has solved such problems successfully in the past might simply need a reminder to check the denominators.

**The Student Model.** The heart of the AnimalWatch system is its "student model" function: a feature that allows the system to estimate what an individual student user understands about the mathematics domain, compare the student knowledge to its expert knowledge of the curriculum, identify appropriate areas of

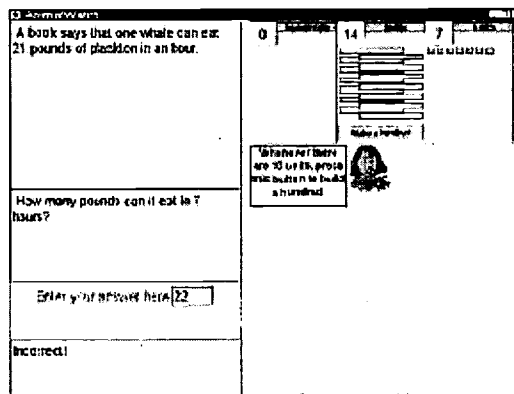


Figure 3: Example of a hint screen

challenge, and construct a specific math problem that is tailored to the student user. The estimate, or "student model", is continually updated based on the student's performance, which is assessed by number and type of errors, response to instruction and hints presented by the system, and latency to reach the correct solution. As the student progresses, hints typically become less concrete and procedural, which encourages the student to generalize past solution strategies and move to a more abstract level of reasoning. The combination of intelligent problem selection and instruction, along with the ability of the system to redirect its teaching focus very quickly in response to student performance, is analogous to having an expert teacher provide individualized tutoring for each student in the class at the same time.

## Evaluation Studies

AnimalWatch has been field tested in three school districts in Western Massachusetts, including both affluent and resource-limited schools in rural and urban settings. The primary dependent measure has been students' math self concept, which includes measures of confidence in one's ability to do math; liking of math; and interest in learning math and the belief that math is valuable to learn (Eccles, Wigfield, Harold, & Blumenfeld, 1993). In pre-post test designs, we have found that 3-6 sessions with AnimalWatch leads to significant increases in math self concept, particularly for girls (Beal, Woolf, & Beck, 1998; Beck, Arroyo, Woolf, & Beal, 1999). The provision of math problems that are appropriately challenging along with the availability of help when errors are made is particularly effective with female students, who are less confident and less interested in math to begin with. In one study, we compared a version of AnimalWatch with the full student model to a drill-and-practice version that simply presented problems in order through the narrative and did not provide the range of hints and help available in the full version (Beck et al., 1999). The results indicated that the "intelligent" version of AnimalWatch fostered more rapid student progress through the curriculum, and that this effect was greater for girls than boys. Consistent with the findings of others, it appears that girls' interest in studying math is more fragile than that of their male classmates; however, responsive and supportive instruction can significantly enhance girls' math self concept.

**Student perceptions of AnimalWatch.** Student responses to post-test surveys indicate that they generally rated the experience of using AnimalWatch highly: Means range from 3.78 to 4.85 (on a 5 point scale) for questions such as "Would you like to use AnimalWatch again?", "When you made errors, did AnimalWatch give you enough help?", "Do you think the computer is a good way to learn math?". On the question, "Did you like working with AnimalWatch?", girls gave significantly higher ratings than boys (mean 4.50 for girls, 4.05 for boys).

Although both boys and girls give high marks to AnimalWatch, they prefer different types of instruction provided by the system, and they respond to different types of instruction. In an experimental study, the system was "tuned" to provide either detailed procedural hints that walked the student through to the correct solution, or more conceptual hints. Girls reported that they liked the procedural hints better, and procedural hints were also more effective for girls (i.e., procedural hints were followed by fewer errors in subsequent problems), whereas there was no relation for boys. The distribution version of AnimalWatch is designed to select procedural or conceptual hints on the basis of the student model's estimate of the student's current comprehension; hints that are highly effective will increase the probability that similar types of hints will be used with subsequent problems.

**Teacher perceptions of AnimalWatch.** During the development of the program, teachers were invited to the UMass campus to try out AnimalWatch and provide feedback. During the workshops, we made an initial presentation about gender equity in SEM education, and outlined the goals of the AnimalWatch project. The participating teachers then worked with AnimalWatch, made notes and suggestions, completed a survey about the system, and participated in a group discussion about its strengths and weaknesses. AnimalWatch received high marks from the teachers, who rated it very highly on such issues as appropriateness of math topics, sufficiency of help, ease of use, fit to their curriculum. They also felt that working with AnimalWatch would help prepare weaker students for high stakes achievement tests such as the PSAT or the MCAS (a new state assessment in Massachusetts). Teachers responded very positively to AnimalWatch's ease of use and resilience (i.e., there is little that a student can do to "mess up" the computer and thus require teacher intervention). They were also very pleased that it will run on any platform. About 75% use Macintosh machines at their schools, but many have PC machines in their homes, and a number reported that the computer labs at their schools included both platforms.

## Educator Support

AnimalWatch has been designed in collaboration with a team of master teachers of 5<sup>th</sup> and 6<sup>th</sup> grade students, and includes several features that facilitate its use in the classroom. First, a student progress report function is provided so that instructors can see at a glance the various curriculum areas that are being worked on by individual students. Second, a companion web site provides background information about the project, support information, updates, and a discussion board. There is also a facility for downloading

the AnimalWatch program “shell”, along with the three adventures. (Due to the large number of images, the entire system is at this point best distributed via CD-ROM unless the user has an extremely fast connection.) Third, the web site offers numerous authoring tools so that teachers can create their own word problems and submit them for inclusion on the AnimalWatch adventures (see Figure 4). Teachers can also create new adventures with their own endangered species.

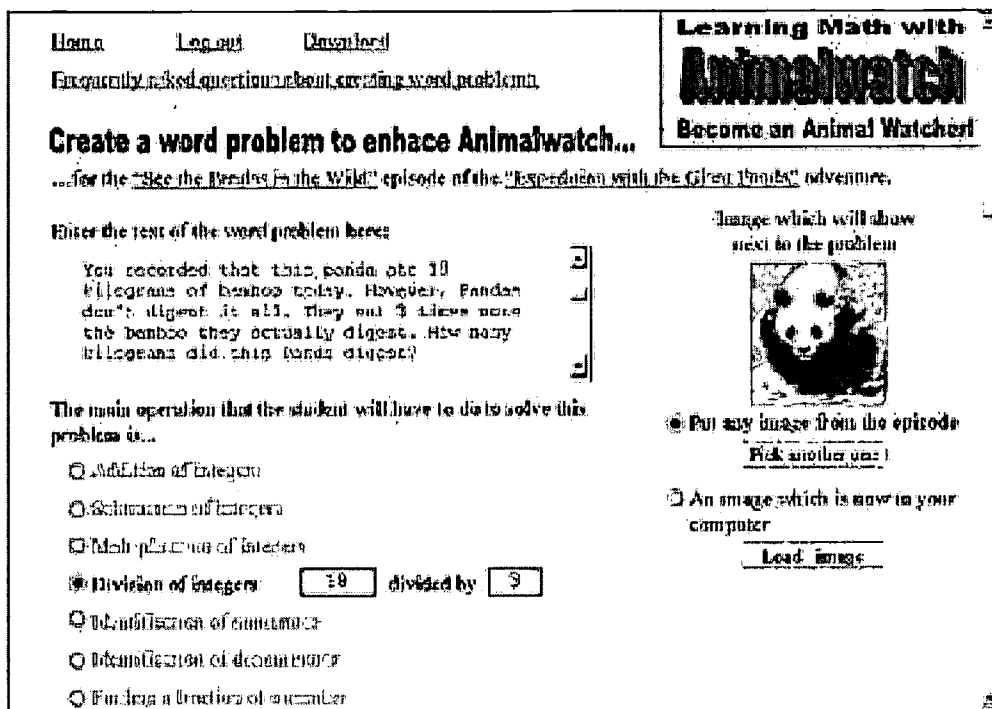


Figure 4: Word problem authoring tool for teachers

## Conclusions

Declines in math interest, as well as gender differences in math self concept, are already apparent by the end of elementary school. Unfortunately, many new teachers at the elementary level, most of whom are female, report that their own mathematics education was weak, that they feel better prepared to teach reading than math, and that they prefer to focus on reading instruction. Thus, there is an urgent need for high quality resources to support mathematics instruction, to encourage all students to feel confident in their ability to learn math, and to maintain student interest in pursuing mathematics training. Many commercial mathematics CD-ROMs for students are simply “drill and practice” packages that do not provide individualized instruction and tutoring; many are also inappropriate for classroom use due to commercial content (e.g., TV cartoon characters), or are targeted primarily to male users (e.g., successful problem solving earns the opportunity to “blast” a target; heavy emphasis on competition).

In contrast, AnimalWatch has been designed to mesh with the classroom curriculum. It uses the same vocabulary and materials that students will see their teachers use during lessons. It links mathematics with an area of the curriculum that both male and female students like: environmental biology. It employs sophisticated artificial intelligence algorithms to provide individualized tutoring to students that is both challenging and supportive. AnimalWatch has been shown to engage student interest and to boost students' math self concept. It is easy to use and is self contained; teachers can allow the system to pace student work. At the same time, it offers teachers the chance to review student progress, and to create new materials when they are ready to do so. Our goal is to reach teacher educators who will be in a position to introduce AnimalWatch to prospective teacher users, and to help us integrate effective technology-based math instruction into as many elementary classrooms as possible.

## **References**

Beal, C. R., Woolf, B. P., & Beck, J. (1998, April). Impact of intelligent computer instruction on girls' math self concept and beliefs in the value of learning math. Poster presented at the annual meeting of the American Educational Research Association, San Diego CA.

Beck, J., Arroyo, I., Woolf, B. P., & Beal, C. R. (1999, July). Affecting self-confidence with an ITS. Proceedings of the Ninth International Conference on Artificial Intelligence in Education, 611-613, Paris.

Eccles, J.S., Wigfield, A., Harold, R.D., & Blumenfeld, P. (1993). Age and gender differences in children's self and task perceptions during elementary school. *Child Development*, 64, 830-847.

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## Prospective K-6 Educators Attitudes about Technology

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Since the late 1970's, the use of technology in the classroom has been increasing at an almost exponential rate. The use of computers and calculators is so prevalent in the modern classroom that it is rarely challenged today, whereas twenty years ago the very idea was practically unthinkable. In that short time frame, most educators and parents have gone from asking themselves whether calculators and computers should be in the classrooms at all to how they may be best used to maximize the learning potential of students.

In the publication "Curriculum and Evaluation Standards for School Mathematics," the National Council of Teachers of Mathematics (1989) stressed the increased use of technology in the classroom at all grade levels. The NCTM also emphasized that students should have access to a computer at all times in their classroom, and that each school should have some type of computer laboratory facility to allow whole groups or classes to work on computers at the same time. In the publication of "Principles and Standards for School Mathematics", the NCTM (2000) asserts that technology has become an essential part of teaching, learning and doing mathematics.

However, the NCTM (2000) also states that technology in and of itself is not a panacea. Calculators and computer are tools, much like pencil, paper and manipulatives. As with any tool, calculators and computers can effectively enhance the student's mathematical learning experience or detract from it. Those experiences depend largely upon how the mathematics teacher uses technology in the classroom. Thus the preparation of mathematics teachers in the proper use of technology is of utmost importance.

The use of technology should not begin in the high school mathematics classroom. Rather, computers and calculators should be used throughout the K-12 learning experience. Many colleges and universities have incorporated courses and programs that include the use of technology as part of the pre-service secondary mathematics teacher's preparation. In many situations, elementary teachers are not given the same preparation or background, which may lead to future elementary teachers concluding that using computers or calculators will not be an important part of their teaching experience.

This paper will present the findings of a work-in-progress study concerning the experiences with technology and attitudes towards technology of pre-service elementary school teachers. In addition, it will focus on what this unique group believes should be the role of technology in the classroom, and how they personally will use technology in the classroom. This information was gathered through the use of surveys during the Fall Semester of 2000. The surveys were given to students at Northern Arizona University, a mid-sized university in the southwest United States. Note: Since the data is still being analyzed, many of the results are still pending; they should be ready by the time this paper is presented during the conference. However, there are some preliminary descriptive results of data analysis of one survey, which will be described and presented below.

Computer attitudes were measured by the Computer Attitude Scale, a forty-item questionnaire developed by Loyd and Gressard first in 1984, and later modified. The Computer Attitude Scale is an instrument that measures attitudes towards learning about and using computers. It is perhaps the most extensively used and tested scale of its type in use (Woodrow, 1991). The developers claim that this scale is "a convenient, reliable, and valid measure of computer attitudes, and that it can be confidently and effectively utilized in research and program evaluation contexts." (Woodrow, 1991) The instrument provides scores on four different scales: Computer Anxiety, Computer Confidence, Computer Liking and Computer Usefulness. Positively and negatively worded statements are included in each of the four areas. Alpha reliability coefficients calculated by the creators of the instrument for each of the four subgroups range from .86-.88, .87-.91, .88-.91, and .82 for each subscale, respectively with the total score having an alpha reliability of .95. An independent researcher found similar reliability coefficients (Woodrow, 1991). Each subscale consists of ten items. Students responded to each item by selecting one of four possible responses; strongly agree, slightly agree, slightly disagree, and strongly disagree. Some sample items from

the CAS are statements such as “Computers make me feel nervous and uncomfortable” or “Computers do not scare me at all”. The correlation between the subscales range from .69 to .84. Scores can range from a low of 10 to a high of 40 on each of the subscales. In general, the higher the score, the more positive the attitude about computers. Any composite score above 90-95 indicates a neutral to positive attitude towards computers. In previous studies, it was found that gender played no significant role in computer attitudes (Loyd and Gressard, 1987), and that computer attitudes were significantly affected by computer experience (Loyd and Gressard, 1984).

The Computer Attitude Scale was administered to four sections of Principles of Mathematics I, the first of two one-semester courses that are required of all elementary education majors at Northern Arizona University. Students in these sections were asked to fill out the survey, but it was not required; a total of 100 students responded to the survey. Of those students, 91 were female and 9 were male; most (56) were freshman, and almost all students (87) were under the age of 22, which is the lowest age category on the Computer Attitude Scale. Surprisingly, a large number (70) indicated having more than one year of experience with or learning about computers; only 25 had less than six months experience. Some of the more common experiences listed by students were using the computer for word processing, for e-mail, and for surfing the World Wide Web.

Results showed that only five of the 100 students had a composite score below a 90, which would indicate that most (95) students indicated a neutral to positive attitude towards computers. Of those five, four had six months or less experience working with and learning about computers, which may have attributed to their negative attitude.

The results from the individual subscales are shown in Table 1.

Subscale	Mean Score	Scores of 40-36	Scores of 35-31	Scores of 30-26	Scores of 25-20	Scores less than 20
Computer Anxiety:	32.10	31	37	19	9	4
Computer Confidence:	29.71	18	31	26	17	8
Computer Liking:	26.62	7	21	29	26	17
Computer Usefulness:	33.28	30	49	19	1	1

Table 1: subscale results of the Computer Attitude Scale

All means indicate that students show a neutral to positive attitude among these four subscales. However, there does exist some variety among the subscales. Students exhibit a relatively strong positive attitude towards the usefulness of computers, yet were not so positive when it came to liking computers. Computer anxiety, unlike mathematics anxiety, does not seem to be a major problem among these students. Also, students overall indicate a fairly positive attitude in their own computer confidence.

The individual numbers in each subscale present some interesting findings as well. The fact that so few students scored above a 30 in the Computer Liking subscale when compared to the Computer Usefulness subscale is of particular interest. This seems to indicate that while many students believe computers are useful, to say that they like computers might be stretching the truth a bit. The fact that almost one in five students scored less than 20 in the Computer Liking subscale (which would indicate a negative attitude) is particularly alarming. However, almost half of the students in that category have less than six months experience working with and learning about computers. This could mean that the more these students experience using the computer, the more positive their attitude would become towards computers. As for those students with negative attitudes that have significant experience with computers, more study is necessary to determine why these students may feel this way.

A quick glance at the individual statement results provides more insight. On the statement "Learning about computers is a waste of time", 97 students either strongly disagreed or slightly disagreed. While this is an excellent result, one must wonder about the three who agreed with the statement. Another finding of particular interest concerned the statement "I'll need a firm mastery of computers for my future work". The results were mixed; fourteen students strongly agreed with the statement and seven strongly disagreed, but the majority either agreed or disagreed only slightly. Of the 10 statements that dealt with Computer Usefulness, this question had the most negative response. Given that these students will be often, as teachers, the first to introduce primary grade children to the educational value of the computer, this result more than any could indicate that they might need more preparation in the use of technology in elementary school. More study into the results of why students responded as they did and the implications of those responses will be undertaken with the results presented during the conference.

References:

Loyd, B.H., and Gressard, C. (1984). The effects of sex, age, and computer experience on computer attitudes, *AEDS Journal*, 18, 67-77.

Loyd, B.H., and Gressard, C. (1987). An investigation of the effects of math anxiety on sex and computer attitudes. *School Science and Mathematics*, 87(2), 125-135.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Woodrow, Janice E. (1991). A comparison of four computer attitude scales. *Journal of Educational Computing Research*, 7(2), 165-187.

# Using Databases in Teaching Advanced Mathematics Courses

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**Abstract:** This paper is a follow up of the papers that discussed how information technologies could be used in teaching Advanced Mathematics (Bouniaev, 1995) and in developing general logic actions (Bouniaev, 1999). Here we discuss the possibility of using databases in teaching Advanced Calculus. The psychological foundations of our considerations are Stage-by-Stage Development of Mental Actions Theory and Constructivism.

## Introduction

This paper is a follow-up of research contained in the paper "Some Psychological Aspects of Developing Computer Based Instruction in Undergraduate Advanced Mathematics" (Bouniaev, 1995) as well as in "Computer Based Instruction and General Logic Action Development In Teaching Mathematics" (Bouniaev, 1999) In these papers, we mainly discussed the methodology of using Information Technology (IT) in studying such disciplines as Calculus and Foundations of Algebra and Analysis. They focused on the development of elementary general logic actions such as classification and attributing to a concept. The psychological foundation of the above-mentioned papers was Stage by Stage Development of Mental Actions Theory (SSDMA theory).

Since the first paper in the series was published, long discussions and collaborations with M. Connell led us to the conclusion that SSDMA prescripts for organizing a learning process practically coincide with those of the constructivism (Connell & Bouniaev, 1996).

Thus we would like to emphasize that although we still explicitly proceed from the concepts and recommendations of the SSDMA theory, implicitly there are no contradictions with the theory and practice of constructivism. Proceeding from the above-mentioned psychological theories as a fundamental basis for using information technologies (IT) in this paper we discuss the possibilities of using databases in organizing instruction process of such an abstract discipline as Advanced Calculus. This discipline is a part of most math teacher training curricula and can substantially impact the efficiency of teaching Calculus in high schools.

In the first part of the paper we give a brief summary of the SSDMA theory, which is essential for understanding the paper. Here we repeat most of what we said about it in (Bouniaev, 1995). In the second part we analyze some particular aspects of using IT in studying more advanced math courses as compared with lower level courses and their significance for math teachers training. And finally proceeding from the SSDMA's five stages of development of mental actions we describe the possibilities of using such databases as Access at the initial stages of action development.

## Basic concepts of the SSDMA theory

This paragraph is based on the monograph of Talysyna (Talysyna, 1975). According to this theory the major goal of instruction is developing mental actions with objects of the studied field. Instruction is viewed as controlling students' activities and hence controlling the process of development. Thus, instruction efficiency is determined to a great extent by a well-developed system of control.

All actions can be referred to two categories: general logic actions and specific actions. General logic actions are inherent in every subject field and are different only in objects at which they are directed.

Examples of such type of actions are qualification; break up into classes, comparison, contributing to a concept, action of proof. For example, qualification as a type of action exists in mathematics (qualification of the conics and differential equations) as well as in other disciplines. Specific actions are basically inherent to a given subject field. For example, in mathematics they are arithmetic operations, differentiation, etc.

The SSDMA theory specifies four independent characteristics of any action used to judge the level of development of an action:

**Form of action.** An action can be in a materialized (material), speech or mental form. A materialized form of action is connected with manual activities (manipulation, hand-on activities, etc.); objects of action (or their models) are presented in a material form; results of action should be real transformations of these objects or of their models. There is no need to discuss the speech form of action. However, it should be noted that according to the SSDMA theory, writing belongs to the same speech form. Mental form of action is the highest form of action development. An action in this form is imperceptible for one's associates and its results are also recorded in an imperceptible for others form. For example, in considering a specific problem a task of qualifying conics may arise, which is a subsidiary problem with respect to the main one. If a number of conics presented for qualification is not excessive then any mathematician can perform this action in his/her head and record its results in memory. As experiments show, for students studying the theory of Conic Sections it is difficult to perform this action in their heads (even if the number of conics at the beginning of study is 3 or 4). The mental form of action also means that its objects are representations, notions, concepts and all operations are performed in the mental form. The ability to perform a whole action in the mental form indicates that it has gone through all the stages of development and interiorization.

- Each of the described forms of actions has another three independent characteristics: (a) degree of generalization; (b) degree of completeness; (c) degree of assimilation.

(a) Generalization of an action means ability to apply it to objects of a different nature. If the degree of generalization is high enough a person can easily apply this action to different objects, or example, if the action of qualification of conics is developed with a high degree of generalization then it can be applied to a similar problem in three-dimensional space. In this case a person has developed some general ideas how to perform a qualification action.

(b) The degree of completeness indicates if all the operations that were to be performed in the process of performing an action have been actually completed. For example, in teaching the action of qualification of animals an operation to be performed is obtaining enough specific characteristics to be able to refer an object to a particular class. It is evident that sometimes all the answers can be correct but based on superficial characteristics which indicates that in the process of performing an action either wrong operations were made, or the order of operations was wrong, or not all of the necessary operations were performed. The common method of establishing the degree of completeness of an action is restoration of all the operations necessary to perform it.

The degree of assimilation, as a rule is connected with such indicators as the speed of performing an action, technical errors and mistakes, the level of automatism, etc. It should be noted that drill-and-practice software first of all is aimed at developing certain actions with a high degree of assimilation.

The SSDMA theory singles out five stages in the process of instruction. Detailed analysis of each of the stages in organizing a computer oriented learning environment is given in "Development of multifunctional dialogue CAT programs " (Bouniaev, 1991), therefore here we will provide just a brief outline of these stages.

The first stage deals with the presentation of the material to students and description of an action to be developed. Note that passive reception of knowledge by students is not performing an action. The second stage consists of developing actions in the materialized form. The third stage deals with developing an action in the speech form (both oral and written). The fourth stage is beginning interiorization. Each of us probably noticed that before performing an action to make it easier we tend to speak it out to ourselves. Speaking out to ourselves is the fourth stage of development. At the fifth, final stage, the action is developed in a mental form. Note that at each of the last four stages the action is developed at a certain given level of generalization, completeness and assimilation.

## Construction of materialized objects in studying advanced mathematics

As it follows from most modern psychological theories, for example, constructivism or SSDMA theory, at the initial stage of instruction the object of action should be presented in a material or materialized form of action (Bouniaev & Connell, 1996). Moreover, this action should be developed at a high enough level of generalization, completeness and assimilation. As a rule it is not hard to do when we deal with development of arithmetic or algebraic actions. But the more abstract the material, the more difficult it is to present the object of action in a material or materialized form. Thus in organizing the instruction of more advanced math courses it is common practice to start right from developing the action in the mental form. This negatively affects the level of student's knowledge in abstract math disciplines.

The math teacher curriculum traditionally includes such disciplines as College Algebra, Calculus, Linear Algebra, Foundations of Algebra and Analysis, Abstract Algebra and Advanced Calculus. In training a math teacher each of the above-mentioned courses plays its particular role. A change of this role in studying more advanced courses significantly affects the character of didactic goals to be achieved in the process of instruction. This change can be easily observed in changes in wording of the exercises of the above-mentioned courses.

Thus in College Algebra and Calculus I the majority of exercises deal with finding a new object. For example, find a sum of two algebraic fractions, find a derivative or an integral, etc. At the final stage of instruction of Calculus and in teaching Linear Algebra the number of problems dealing with finding new objects is balanced by the number of classification or attributing to the concept problems. For example, to prove that the given series converges (in Calculus), or to prove that the given system of vectors forms a basis (in Linear Algebra). So we see a new type of problems, however, as we already mentioned they deal with development of elementary logic actions.

As a rule, the object of action of this type of problems can be easily presented in the material or materialized form. For example, it is easy to represent the terms of converging series as a sequence of dots on the plane. However, it should be noted that a model presentation of an abstract notion that looks superficially good is not always helpful in doing a specific problem.

Development of logic actions such as classification is connected with construction of materialized models that physically may not look like the respective actions but present in the materialized form the structure of such actions. For instance, tables can be used that students fill in while doing specific problems. It is also expedient to use electronic text fragments, which can be moved to different places on the screen. The details of using IT in solving this type of problem can be found in (Bouniaev, 1999). Therefore we are not going to elaborate it here.

At the next stage in teaching such courses as Advanced Calculus and Abstract Algebra besides simplest logical actions such as classification and attributing to the concept we have to develop more complicated logic actions such as the action of proof. Bearing in mind that in studying Calculus students have mastered the simplest "transformation problems" like finding a derivative or an integral, then in Advanced Calculus development of the action of proof becomes the major goal of instruction. And not only development of the action of proof, but proof of statements applicable to the whole class or classes of objects. In this case the object of action is a theoretical abstract concept, which makes it significantly different from the situation when the object of action is a particular function even if the action itself is an action of proof.

Thus for example the task to prove that function  $\sin x$  is continuous is significantly different from the one to prove that any continuous on the closed interval function is bounded. In the first case the object of action has materialized representation both in the form of a formula connected with numerous trigonometric identities and in the form of a graphic using which all the actions performed in the course of proof can be "materialized". In the second case the object of action is a math abstract notion for which it is difficult to find a materialized representation. Any continuous function viewed as an illustration is just an example of an object coming under this notion and thus cannot claim to be a materialized representation of a math abstract notion.

Besides that, math abstract notion itself significantly depends on the level of development of an individual in whose mind it exists. For example, the notion of continuity exists both in the mind of a freshman before studying Calculus I and in the mind of a senior ready to study Advanced Calculus. At the same time in the framework of the studied course (Calculus or Advanced Calculus) further development of

abstract notions determines different actions that are objects of these abstract concepts.

For example, in Calculus I to develop the notion of continuous function it is enough to use the action of classification of pictures illustrating graphics of continuous and discontinuous functions. We can also effectively use pictures in a formal proof of continuity. However, in the proof of theorems in which continuity is a premise or a result, an illustration may help but it cannot serve as a materialized model of proof. A model should reflect not a particular case of the proof of continuity but general characteristics of any proof. In the framework of the model needed for instruction, "proof" and "proof of continuity" are related as general and particular. As the conducted experiments showed databases or bases of knowledge can serve as materialized models in teaching proofs.

## Databases in the Study of Advanced Calculus

A natural information model of any subject field including mathematical theories is a database or a base of knowledge. We consider databases a named complex of data reflecting a stage of an object or a number of objects, their properties and relationships. In fact a database can be viewed as an information model of a given object, and effectiveness of the control system of the object depends on its preciseness and authenticity.

Using databases or bases of knowledge in the process of instruction is not something new. Already ten years ago reference and information systems built within the framework of prevalent databases constituted about one third of all the databases used in teaching math. However, we offer much more than just using databases for reference and information purposes although we do not exclude it either. Let us consider the possibilities of using databases at every of the five recommended by SSDMA theory stages of developing mental actions based on the organized process of instruction of Advanced Calculus.

According to the SSDMA theory at the first stage of instruction a student gets necessary information about a goal of action and its objects. The instructor in familiar from previous courses terms explains a system of reference points (prompts that help the student to master the studied material.) He/she also discusses the content of the studied material. It should be mentioned that even at this early stage of instruction, certain difficulties arise first of all because it is hard to explain to a student what the goal and object of instruction are. The thing is that students have been studying Calculus for three semesters but they still have a very vague idea of theoretical math fundamentals thus making it very difficult for them to understand what kind of proof they are going to deal with in this course. For the majority of students at this level, math is associated first of all with such words as find, calculate, etc. Problems of existence and therefore of proof are still completely alien for them. It becomes even more confusing since in most traditional advanced calculus textbooks the problems (especially in the beginning) are worded in the same way as they are formulated in their Calculus textbooks.

Questionnaires to monitor the students response that we have been distributing and analyzing during the last ten years of teaching Advanced Calculus invariably indicate that even after two or three weeks of instruction students were confused about what is required from them to succeed. At the same time most Advanced Calculus students already took courses incorporating databases. Thus terms associated with them such as table, record, form, report, query, relationship, etc., are familiar to them and do not provoke psychological rejection.

Using this previous experience with databases as an advantage we determine the goals of instruction as structuring the material contained in the textbook and explained in class. By structuring we mean constructing databases using the studied material of the course. Naturally the structure of any database significantly depends on what kind of information this database is used for. Therefore as points of reference for constructing databases we offer students typical questions that will be part of the test. Naturally, the most important points of reference are the examples of proofs discussed in class. By no means we imply by this that this is enough to teach students the action of proof. At the same time we include into the system of prompts questions like: "how to build a database so that using it one could reproduce the proof presented in class?" We also linked questions that students had to answer using their own constructed databases to the points of references. It was mentioned that the tests would include the following questions.

1. Formulate the theorem.
2. What is the theorem's premise?
3. What is its conclusion?
4. What notions are used in the theorem's formulation and what is their meaning?
5. What theorems were used in the proof of the given theorem?
6. Give examples of problems, which are based on solving this theorem.
7. What

theorems need to be used to do this problem? 8. Is the conclusion true in absence of any of the premises? 9. Are all the premises necessary for the conclusion to be true? 10. If you answered, "yes" in 9, demonstrate it. 11. Give an example of an object that can be attributed to this concept. 12. Give an example of an object that cannot be attributed to this concept. 13. Prove this theorem. 14. Solve a proof problem.

It should be noted that at this stage of instruction students are not yet involved in performing the action or they perform it in a perceptive form, i.e. students just observe how it is performed. Besides, in the process of demonstration /observation points of reference (prompts) are determined that help to develop the action at the next stages. At this stage the instructor can also use his/ her own prepared database as an information system to get illustrations of examples, links with respective sites in the Internet, etc. Certain fields in such a database can be "objects" representing different program packages. At the second stage proofs are developed in the materialized form. In the course of experiment the entire group (about 30 students) was divided into subgroups of three. Students took decisions concerning the structure and elements of databases independently. It was assumed that any member of the group could use this database during the test.

At the same time there were certain limitations to the base itself and ways of its use. First, none of the fields could contain more than 50 symbols; second, none of the fields should contain a statement that could be divided into two conjunctive statements. For example, none of the fields could contain the statement "continuous on the closed interval function". Such field should be broken up into three: a) function domain is interval; b) function is continuous; c). interval is closed. This way the students could not record the whole theorem in one field. This requirement also makes students seriously contemplate the possibilities of structuring any material.

The next set of requirements deals with the possibility of using databases during tests. First, students should act quite fast since the number of problems presupposed certain operational speed. Second, to speed up the process, the students could print out certain forms from databases but only those that closely followed the question in the premise of the problem and contained no additional information. That demonstrates how at the second stage of developing the action of proof we used the materialized form to enhance the instruction process. Students worked with texts and databases. The texts were given but they constructed the databases independently. We also focused on all the characteristics of developing the action of proof in the materialized form to ensure its success – generalization, completeness, and assimilation.

A high enough degree of generalization was achieved by involving students in the action of determining for themselves fields of records in a universal way so that they accommodate analysis of all other theorems as well. If the fields of record determined for one group didn't accommodate a new theorem then the whole group had to be reviewed. A high degree of completeness was provided by the process when all the activities were broken up into two classes - determining the structure of databases and creation of new records, queries, filters and relationships. The first part implied breaking up an action into all the included operations and the second enabled its performance in the automatic regime. And finally, a high degree of assimilation was provided by the necessity to structure quite a lot of instructional information. Besides, one of the factors of successful use of databases was the speed of acquiring and utilizing necessary information.

As we already noted, the students themselves determined the structure of databases, fields, records, etc. However, at the beginning of this experiment we had a certain database structure in mind. First of all since we deal with development of the action of proof, it seems expedient to have a table of theorems, with the following fields: "Name of the theorem" (for example "Intermediate value theorem"); "Object of the theorem" (for example "A function  $f$ "); "Premise 1" (for example "Domain is an interval  $\langle a, b \rangle$ "); "Premise 2" (for example "Continuous on the interval"); "Premise 3" (for example "Domain is a closed set"); "Premise 4" (For example " $f(a) < C < f(b)$ "); "Premise N"; "Conclusion 1" (for example "There is a point "c" between "a" and "b" such that  $f(c) = C$ ); ...; "Conclusion N"; "Concept 1 used in the theorem statement" (for example "Continuity on a set"); ...; "Concept N used in the theorem statement"; "Model Problem 1"; ...; "Model Problem N".

It also seems useful to have a table "Theorem Proofs" including the following fields: "Name of the Theorem"; "Object of the Theorem"; "Definition of Concept 1 used in the proof"; "Definition of Concept N used in the proof"; "Theorem 1 used in the proof"; ... "Theorem N used in the proof"; "Transformation 1 of the proof"; ... "Transformation N of the proof"; "Proof structure".

We also believe it would be helpful to create such tables as "Concepts and their definitions", "Model Problems", etc. Creating queries plays an important role in the instruction process. As with tables the students make decisions what queries to create independently. It looks expedient to create queries that provide access to all the records (or their parts) that have the same conclusion; like "sequences converge" It is also useful to create queries



enabling to find a model problem solution proceeding from the problem being solved.

The third and fourth stages of action devolvement are designed to develop an action in external and internal speech forms. As we already noted, results of students activity at this stage should be either articulated out loud or written down. If at the previous stages in organizing the instruction process we focused on determining the structure of databases and building them then at this stage it already played a secondary role. At the same time, naturally, in the course of study new records were created and added. As a task, the students were required more and more often to write a standard proof using information from the student databases. If in the first test questions 1-12 were prevalent, then in the second test we included questions immediately related to proofs (13-14) and the third test consisted mostly of proofs with access to databases.

At the last stage the action should be developed in the mental form. At this stage students continued to replenish their databases, however, their use as an information system was limited to an emergency. Using databases during the test was penalized, though insignificantly.

## Conclusions

Recommendations proceeding from the theory of constructivism and SSDMA theory in studying abstract math disciplines can be formalized using databases. The database approach is hard to overestimate in forming actions in the materialized form. At the same time using databases proved extremely effective at all the five stages prescribed by the SSDMA theory.

## References

- Bouniaev, M. (1991). *Development of multifunctional dialogue CAT programs*. Moscow: Publishing House Prometey.
- Bouniaev, M. (1995). Some psychological aspects of developing computer based instruction in undergraduate advanced mathematics. In *International Conference on Computers in Education, Proceedings of UCCE 95*, Singapore; December 5-8,1995, (pp.583-591). Charlottesville, VA: Association for the Advancement of Computing in Education.
- Bouniaev, M. (1996) Stage-by-Stage Development of Mental Actions and Computer Based Instruction. In B. Robin, J.D.Price, J.Willis, & D.A. Willis (Eds.), *Technology and Teacher Education Annual* (pp.947-951). Charlottesville, VA: Association for the Advancement of Computing in Education.
- Bouniaev, M (1999). Computer Based Instruction and General Logic Action Development In Teaching Mathematics. In David A. Thomas (Ed.), *International Conference on Mathematics/ Science Education & Technology 99, Proceedings of M/SET 99 - March 1-4, 1999, San Antonio, Texas*, 211-215, Association for the Advancement of Computing in Education
- Connell M. & Bouniaev, M. (1996) Constructivism and SSDMA - Implications of Two Learning Theories on Technology Use in Mathematics Teacher Education. In B. Robin, J.D.Price, J.Willis, & D.A. Willis (Eds.), *Technology and Teacher Education Annual* (pp.202-208). Charlottesville, VA: Association for the Advancement of Computing in Education.
- Galperin, P. & Talizyna, T. (1979) Contemporary condition of SSDMA theory. In *Vestnik MGU, V.14, #4 Psychology* (p p.54-63).
- Leontyev, A.N.(1972). *Problems of psychological development*. Moscow: Moscow State University Publishing House.
- Talizyna, T. (1975). *Control of a knowledge acquisition process*. Moscow: Mosc. St. Univ. Publishing House.

# Using Excel to Explore a Thematic Mathematics Unit: Ideas, Reflections and Suggestions

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**Abstract.** The fifth grade mathematics curriculum at the University School of Indiana University of PA contained a long-term project on the real-world concept of budgeting. Students were to develop a budget to spend \$1 million on a specific interest. Learning to track project expenditures in a spreadsheet was a natural next step. Using PowerPoint to communicate the results added another technological dimension and integrated language arts. Along the way, students learned financial concepts and reviewed mathematical concepts in a real-world context and in a way that supports NCTM standards. As they gained confidence with Excel, students shared their own discoveries. The instructors gained valuable reflections about what works in infusing technology into the curriculum, and the project proved the efficacy of Donald Schoen's model of the reflective practitioner.

## Introduction

The fifth grade mathematics curriculum at the University School of Indiana University of PA contained a long-term project to enable students to experience the real world concept of budgeting. Students developed a budget to spend \$1 million on a specific project or interest. Learning to track project expenditures on a spreadsheet was a natural next step. The use of PowerPoint to communicate the results to classmates added yet another technological and curricular dimension. The University School is the campus laboratory school, where curricular ideas like these can be developed, field tested and disseminated. This project came about in Spring 2000 when two faculty members became involved in a professional development model created in a PT3 Grant. The objectives of the million-dollar Excel project addressed both content and technology. Students were to see how mathematics is used in their daily lives and learn the fundamentals of budgeting. They were also to learn the basics of the MS Excel program and refine their skill in using PowerPoint as a communications tool. Finally, the project addressed Pennsylvania Standard 3.7.7, Grade 7 D—apply computer software to solve specific problems: i) identify software designed to meet specific needs; ii) Identify basic multimedia applications; iii) Demonstrate basic knowledge of desktop publishing applications.

## Methodology and Results

After looking at examples, the class was able to develop an operational definition of a spreadsheet: a table created with computer software; contains information, especially numbers; does calculations for you; often used for spending plans; can answer "what if" questions – e.g., *what* could I buy *if* my allowance went up, *if* the cost of a computer game went down, or *if* taxes went up. Among the examples were small budgets such as an allowance, as well as a more elaborate million-dollar budget. They also saw three 'what-if' scenarios, demonstrating the impact of price changes on one's allowance.

Students then got to work on a "Puppy Budget," where their task was to plan what they would need to acquire and keep a pet for a year, using MS Excel. The simple initial budget contained only a purchase column, a cost column and a total. With little urging, students soon forged ahead in adding formatting. Their first function was AutoSum, which they pronounced "awesome." Next we added columns for unit price, total cost

per item, “must have” items, and a “what’s left” column or balance. It was now time to learn formulas for multiplication, subtraction and adding. Requiring more work, formulas were not quite as awesome as AutoSum, but the ability to copy rather than type them saved the day.

Our class was now ready to apply their Excel skills to the Million-Dollar Budget. They brainstormed with partners what to buy, entered a list of planned purchases into Excel, looked for prices on the Internet and entered them into their spreadsheets, and calculated costs and a running balance in the spreadsheet. A large spreadsheet requires more formatting to be understandable, and students returned enthusiastically to beautifying theirs. They began to discern the difference between “cool” and “understandable” or “legible,” however.

At the next class, students reviewed percentages. Students added columns to show the effect of price hikes and to calculate sales tax. Now that the data were in, we could turn our attention to charts to provide a graphic illustration of where their money was going and the percentage spent on various categories. With many ways to customize, charts proved to be another fertile ground for student creativity. But they learned again that what looked glorious on screen was not always readable in print or when displayed on a projector.

Finally, the students reviewed PowerPoint and created a short presentation summarizing what they would buy and why, incorporating copies of their spreadsheets and charts as well as illustrations from clipart and the Internet. They then delivered these presentations to their peers.

Along the way, students learned financial concepts such as budget and unit price. They reviewed percentages, means, and other mathematical concepts in a way that supports NCTM standards and applied them in the real-world context of budgeting. As they gained confidence, students made their own discoveries and even began to demonstrate them to the whole class. Some things they learned the hard way. The students found it tedious to rearrange their spreadsheets in broad categories and calculate subtotals. When they inserted detailed spreadsheets and charts into PowerPoint, however, they realized their beautiful creations were too large to display effectively on screen. Now they understood that broad categories and subtotals would have been useful. Students often get caught up in the bells and whistles of computer programs. This experience was a first step in learning to use computers effectively—i.e., as a tool that aids understanding and communication.

Dr. Brzycki and Dr. Hechtman also shared some interesting reflections while they planned. Students need a project they can relate to in order to buy into using the Excel program as a learning tool. Models are helpful. Student partnering is not a good method in a project that requires skills acquisition. It is easier to have students hone estimation skills than to determine costs to the penny. This, too, is a real-world lesson in that it encapsulates the difference between accounting and budgeting. It was clear that there were many possibilities for extending the project, and some are already under way—recording and analyzing weather data, Olympic medals, election polls, and states data; and importing data from the web. Spreadsheets are a versatile tool that can serve as a mindtool in many disciplines (Jonassen 2000).

The model of technology diffusion and training used here was a success. A technology expert and a content expert collaborate to design the units. The technology expert leads the first round of classes while the teacher assists; then the teacher leads the class while the technology expert assists; and lastly the teacher can solo. Here Dr. Hechtman gained experience and became enthusiastic about teaching the unit herself. This model was based on the work of Donald Schoen (1988) and is also being used in the university setting in the teacher preparation curriculum at IUP and Clarion University of PA. A PT3 grant (Preparing Tomorrow’s Teachers to use Technology) funds an instructor and/or graduate assistant as technology expert. The model permits both teacher and students to gain or consolidate skills without creating dependence on the expert and encourages colleagues to engage in collaboration and mentoring.

## References

- Adams, E. C. (2000). Transparent Training and Technological Intuition. *T.H.E. Journal*. 27 (9), 115-117.
- Jonassen, D. H. *Computers as Mindtools for Schools*. Upper Saddle River, New Jersey: Prentice-Hall, Inc.
- Schoen, D. (1988). *Educating the Reflective Practitioner*. San Francisco, CA: Jossey-Bass Publishers.

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## Mathematics Teachers on Track with Technology

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**Abstract:** It is a challenge for teachers at all levels to bring mathematical concepts alive for their students, and to engender excitement about the technology tools used to facilitate data representation and communication. We introduce a teacher training process in which participants actively engage in investigations that integrate information technologies into the teaching and learning of mathematics. These investigations can be used in both teacher training and in the classroom. Our approach has several advantages: Mathematical concepts are introduced through relevant technology-enhanced problem investigations, engaging participants in defining and solving problems. Group interactions promote creativity and variety in ideas and approaches. Participants learn to present ideas in computer-based, multi-media formats. In addition, the training process is a model for the active, small-group learning process that students will experience, and is thus an experience in modeling for teachers. The process also prepares participants to formulate similar instructional techniques of their own.

### Introduction

Today's students are facing a world that is becoming increasingly information-driven in both work and the home, creating a more urgent need for all students to develop a clear understanding of mathematical principles and the ability to use technology in communication and problem-solving. In the workplace, students will also be expected to work in teams to solve problems (Felder & Brent 1996), which may not already have known solutions, a very different situation than the traditional math-class problem. And yet, researchers increasingly agree that this type of engaged, active learning, which occurs when students actively participate to learn concepts in a meaningful context, is important in building student comprehension, retention, and internalization (Jones, et al. 1994; Felder & Brent 1996). Girls on Track aims to design and disseminate technology-enhanced, student-centered investigations of urban and social problems to join the strengths of engaged, meaningful learning, with the tools of technology, to help students develop strong, creative skills in problem-solving, mathematics, and data gathering, representation, interpretation, and presentation.

Girls on Track is a multi-institutional program that was developed to encourage middle school girls to continue taking advanced math and computer science classes through high school by introducing girls, educators, and counselors to new ways of approaching mathematics and technology. This program could not be implemented without the foresight of the principal investigators and the help of a dedicated staff, whom you will find listed in our program report (Berenson, et al. 2000). In this paper, we describe the teacher training and investigation aspects of the Girls on Track program, which we believe will be beneficial to educators in all math and science related areas. Our findings are preliminary, but we believe that our methods are supported by educational research theory. In the first section, we describe the summer program in brief. We then discuss the details of our investigation method, how it is introduced to teachers, and how teachers may use the method in their own investigations. In the final section, we reiterate our findings and discuss future plans for the program.

### The Program

During the summers of 1999 and 2000, Girls on Track, our 3 weeks of professional development workshops and summer camp introduced two groups of middle school teachers, guidance counselors, pre-service teachers, graduate students, and middle school girls to mathematics and computing in an integrated, small-group learning process centered on primarily math and computer-based investigations of local and national urban and social issues. We designed several investigations for Algebra I that integrate group learning techniques, and information search, gathering, and presentation, all facilitated by the use of our website. We began each investigation with a group brainstorming exercise on a central question like, "What will life be like in our county in the year 2020?" Each team generated their own predictions, and moved to the computer labs where our web page reiterated the brainstorm question, provided links to web sites containing relevant data and provided a direct link to a blank PowerPoint template, which each group down-loaded and modified to create their own presentation. Every presentation was loaded onto our website, which was then accessed to download the slides for presentations to the whole group.

The summer program began with a full week of professional development for middle school math teachers, guidance counselors, pre-service teachers, and graduate students, all of whom we will refer to as camp counselors or simply counselors. Camp counselors were broken into groups of 3-5 professionals, and were presented with our math investigations in the same manner that girls would experience the investigations in the following two weeks. Each group created their own interpretations of each problem we investigated, including topics such as population growth, solid waste disposal and reduction, traffic congestion, and the relationship of math skills to job salaries. Counselors experienced first-hand the excitement and difficulties which the students would later face. Rising to the challenge, each group formulated a problem statement, collected and interpreted data, and presented their findings.

In the course of learning this new investigation technique, counselors also were introduced to new technologies, including Internet browsers (Netscape) and searching (via [www.google.com](http://www.google.com)), Excel spreadsheets and graphs, PowerPoint presentation software, and FTP. Each of these technologies was introduced on an as-needed basis during the investigation process, with scaffolding to facilitate their use (see Soloway, et al. 1996). Computers were initially set up with browsers ready and on the appropriate page on our program website, [ontrack.ncsu.edu](http://ontrack.ncsu.edu). This page reiterated the investigation questions, gave links to relevant data on the Internet, and also linked to Excel and PowerPoint templates and help sheets. Each progressive investigation web page linked to fewer templates and offered less guidance in technology use, to promote participants to develop their skills in using these tools. This scaffolding prevents the need for educators to become experts in all the technology they use, instead providing the tools to get started, like "handles" for both professionals and students, leaving the door open for more exploration when creativity drives participants to look for new ways to use the tools.

After the first week of professional development, the summer camp began, with sessions in the morning where professionals would act as counselors for middle school girls, joining them in their own investigations of the same problems. As group facilitators for the students, professionals had the chance to apply what they'd just learned about this new pedagogy. This is a crucial aspect of Girls on Track – without this immediate application of new concepts, many professionals would feel under-qualified to bring these same techniques to the classroom. Fear of technology, and the fear of not being an expert in every problem the students face are very strong deterrents to trying new classroom methods. As leaders in the math camp, professionals became aware that this method encouraged both students and leaders to become adventurous explorers in problem solving, working together. This is a sharp contrast with the traditional, unrealistic model of the all-knowing teacher, which makes both students and teachers uncomfortable. Instead, leaders become participants and guides in investigations, and students become peers in problem solving.

## Investigations

Experts agree that using multiple representations and problem solving approaches increases mathematical problem solving ability (Jiang & McClintock, 2000; NCTM, 2000). We believe that the creation of such multiple representations and approaches can be greatly enhanced by the use of technology. During the summer professional development program, investigations are conducted in an atmosphere of collaborative learning where multiple pathways to solutions via technology are encouraged. The methodology of the investigations involves four main phases: an exploration phase, where small groups of problem solvers define their specific investigation question and make their data collection plans; a data collection phase, where the problem solvers collect data via the worldwide web or other methods; an analysis phase, where the problem

solvers use Excel to represent and analyze their data; and an interpretive phase, where the problem solvers design and share a presentation using PowerPoint and other dynamic presentation techniques, some of which have included skits, video segments, and audience quizzes. In line with other technology teacher preparation programs, the use of technology for the mathematical investigations was not for the purpose of teaching technology, "but for the purpose of enhancing mathematics teaching and learning with technology" (Garafalo, Drier, Harper, Timmerman & Shockey, 2000, p. 68).

By design, the Girls on Track investigations follow the same format, and also connect via a common thread. Participants first brainstorm about what problems our local community will face 20 years in the future. Groups quickly realize that population growth is a central determinant of social problems of the future, from overpopulation, to traffic congestion, to waste generation and disposal. These are the very topics of three of our investigations. As an example, our population investigation is illustrated in the following section.

### **Population Investigation**

The investigation is introduced by initiating a discussion about community problems of the future. After problem solvers have shared their ideas with the whole group and realized that understanding population growth is a major factor in solving other community problems, they are ready to work in their small groups to determine how they want to tackle the population growth problem. They must decide how they will collect and organize their data and are provided with several web sites that contain information about population to aid in their data collection. Once they have data in hand, problem solvers used Excel to represent the data in a table and as a graph. We asked that each group come up with some type of mathematical model for their data that they could use to project future populations. Requiring the creation of a model forced problem solvers to think about issues concerning rate of change and inexactness of solutions to real world problems.

### **Teacher Preparation**

Preparing teachers to use technology to enhance mathematical teaching and learning has been considered a major challenge for teacher educators (Mergendoller, 1994). One of the goals of the professional development program is to encourage this use of technology in the classroom and make the implementation as simple as possible. The approach taken during Girls on Track is to offer teachers one week of professional development before the students arrive and then ongoing professional development during the afternoons of the two-week camp for students. Teachers experience the investigations during the first week of professional development exactly how the students are intended to experience the investigations during the summer camp. They come out of the first week of professional development ready to challenge the students and prepared for technical and mathematical issues that may arise. Some may feel a little uncomfortable due to the alternative approach to professional development that we take, but by the end of the first week of professional development, most are very pleased with the results, as indicated by one teacher's statement:

I really appreciate the opportunity I have been given to stretch myself professionally. Although my anxiety level has seesawed since Monday morning, I still feel much better about next week than I thought I would by this time.

This confidence comes for the teachers not only through their experiences working through the investigations, but also by knowing that there is a support system available to them at all times during the camp. In essence, teachers get an opportunity to merge theory with practice, actively building new knowledge as they try out new ways of teaching and motivating students to learn mathematics within a "safe" environment.

Teacher preparation is also greatly enhanced through their collaboration with other educators. Educators experience first hand the joys and challenges of group work, and are better prepared to guide students as they face their own challenges in working together. In addition, as they work together to solve investigation and group interaction problems, the varied skills of each professional provide new insights for all into classroom strategies. Teachers can share different approaches they have used before; what works and what doesn't, and also have the opportunity to share their greatest teaching triumphs with their peers. This interaction truly enriches the professional development experience, creating collaboration among professionals that can enhance their career development for many years to come.

I have really enjoyed getting to know the people on my team and have appreciated their areas of expertise as well as their empathy for my anxiety at times. For the most part, we were very open about our levels of anxiety, learned to depend on each other, and take risks at learning something new.

After experiencing the investigations themselves, teachers are well prepared to create new investigations. The process that we have used to create investigations is quite simple. First, determine a social problem or issue that will be of interest to the problem solvers. (We've used population growth, trash accumulation, traffic congestion, and job salaries, thus far.) The next step is finding resources on the World Wide Web that can provide you with appropriate data, but keep in mind that it is perfectly reasonable to supplement these resources with other data collection techniques. (Our problem solvers were out counting cars near a busy highway for the traffic investigation.) Once data is obtained, the investigation designer should experiment with multiple representations and solutions that are appropriate for the target audience. This experimentation prepares the instructor for possible problems that may arise during the investigation process, ensures that it can lead to appropriate mathematical thinking, and aides in the development of useful scaffolding techniques that may be required in assisting problem solvers throughout the investigation process. One teacher's summer camp 2000 reflections includes the following plans for her own investigation:

I really like being able to use PowerPoint to have students present any of the skills we are learning in class. ... I also have an activity that requires the kids to take a conventional cereal box and redesign it as a cubical cereal box with the same volume. (It shows that volume can remain constant and the surface area changes) Next year I hope to take them to the lab and input the original dimensions of their cereal box into a spreadsheet, put in the formulas to compute surface area, volume, the edge of a cube with the same volume, [and] the surface area of the cube. I would then have different pairs of kids graph different variables to see if there was any relationship. They would then go to a word processing page, explain the relationships they noticed, and paste the graph into that page.

## Conclusions

We believe that Girls on Track has developed a strong, multi-faceted approach to training teachers in technology-enhanced student-centered instructional techniques. Our investigation design garners the benefits of meaningful, engaged collaborative learning, enhancing teacher and student motivation and excitement. Our design also incorporates the use of technology in appropriate ways that encourage creativity and individual exploration. Our training method uses the same instructional techniques we are promoting to engage educators in learning a new pedagogy for their classrooms. As professionals, educators build skills in collaborating with their peers, facilitating student investigations, and designing their own investigations. Our program fosters both independence and interdependence in professionals, offering tools to create new classroom activities, and bringing a network of professionals together to share their knowledge. In addition, Girls on Track enhanced teacher confidence in using technology and in exploring new ideas.

Computers and the Internet are vitally important aspects of this program. Daily schedules online kept everyone up-to-date with last-minute changes, and linked to each activity during the day. Handouts for information technology help were online and available at all times. Investigation guidance, data links, and starting templates smoothed the data gathering and representation. Training in information technologies centered around student tasks, and the creation and expression of ideas, instead of on the applications themselves. Girls and educators were given the opportunity for individual expression and exploration in an open, accepting atmosphere, where ideas were created and shared at impressive rates, and with outstanding results.

This program is evidence that the worldwide web can become a vital part of any education process, enabling creativity and facilitating information gathering and sharing. Both girls and camp counselors created web pages of their own, linking to their own presentations, which could all be accessed through a central location for easy sharing. Our investigations remain on our website ([ontrack.ncsu.edu](http://ontrack.ncsu.edu)), providing resources for teachers everywhere, and serving as a template for the development of other investigations which integrate curriculum material and the use of information technologies.

In a short time, both teachers and girls became adept in communication through the website. Computers complemented the learning process, giving teachers and girls the confidence they need to continue using computer technologies in the future.

We believe we have successfully met our end goals: to boost confidence and motivation of girls, counselors, and teachers in math, teamwork, sharing ideas, and in using technology to achieve these goals. In program evaluations, both teachers and girls expressed increased confidence in using technology. Educators expressed excitement in the program and in using similar investigations in their classrooms.

Girls on Track has built similar enthusiasm in all its participants, encouraging some to try new software, and new classroom presentation styles and methods. Our investigations have given participants firsthand experience in using any available tools to creatively solve problems and communicate their results. When computers and the Internet are not available, any tools on hand can be used, substituting data collection for web searching, and creating poster or slide presentations instead of using PowerPoint. The result is still the same: investigating relevant issues, using problem solving and analysis techniques, and communicating findings builds confidence and motivation in students and educators alike. In the words of one Girls on Track 2000 pre-service teacher:

When my friends and family asked me, "What is Girls on Track?" I told them that it is a math and technology camp for middle school girls. After the one week of staff development, my description has changed. I would still say that GoT is a math and technology program for middle school girls, but also a confidence-builder for young women, a community awareness builder, an environmental awareness builder, a computer technology and math camp for counselors, and a wonderful social experience for all. My computer competencies have grown this week as well as my friendships among fellow educators. I only hope that the girls get as much out of this experience as I have.

## References

- Berenson, S., Vouk, M., and Robinson, T. (2000). *Annual GoT Report (2), July 2000*. Available: [http://ontrack.ncsu.edu/GoT/Documents/GoT\\_Report2\\_9813902.pdf](http://ontrack.ncsu.edu/GoT/Documents/GoT_Report2_9813902.pdf). [2000, Nov. 3]
- Felder, R. & Brent, R. (1996). Navigating the bumpy road to student-centered instruction. *College Teaching*, 44(2), 43-47. Available: <http://www2.ncsu.edu/unity/lockers/users/f/felder/public/Papers/Resist.html>. [2000, Nov. 30].
- Garofalo, J., Drier, H., Harper, S., Timmerman, M.A., & Shockey, T. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. *Contemporary Issues in Technology and Teacher Education*, 1(1), 66-88.
- Jiang, Z., & McClintock, E. (2000). Multiple approaches to problem solving and the use of technology. *Journal of Computers in Mathematics and Science Teaching*, 19(1), 7-20.
- Jones, B., Valdez, G. Nowakowski, J., & Rasussen, C. (1994). *Designing Learning and Technology for Educational Reform*. Oak Brook, IL: North Central Regional Educational Laboratory.
- Mergendoller, J. R. (1994). The Curry School of Education, University of Virginia. In *Exemplary approaches to training teachers to use technology, vol. 1: Case studies* (pp. 4.1-4.24). Novato, CA: Beryl Buck Institute for Education.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Soloway, E., Jackson, S., Klein, J., Quintana, C., Reed, J., Spitulnik, J., Stratford, S., Studer, S., Jul, S., Eng, J., & Scala, N. (1996). Learning Theory in Practice: Case Studies of Learner-Centered Design. [Online]. *Common Ground: The CHI 96 Electronic Proceedings*. Association for Computing Machinery's Special Interest Group on Computer-Human Interaction, Vancouver, British Columbia, Canada. Available: <http://www1.acm.org/sigs/sigchi/chi96/proceedings/>. [2000, Nov. 30].

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## Student Satisfaction with Online Math Courses and Its Impact on Enrollments

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### Abstract

Convenience is the most important benefit perceived by online students of the asynchronous online delivery format. Students appreciate the flexibility of course access at any time and in the comfort of their homes or at the office. That online courses are well received by Golden Gate University students is not surprising, as our student body is predominately composed of part time students who work full time.

Three years ago the math faculty was asked to put a number of courses online. Anticipating difficulties, the math faculty worked hard to design courses that somehow transcend these problems. In the initial stages of development we feared that the online popularity would not extend to graphics-based courses such as those typically taught in the Math department. How would the student draw a graph or write a formula over the Internet without first learning sophisticated software packages? How would they communicate with faculty and classmates other without this capability?

In this paper we will present enrollment trends of online math classes verses traditionally delivered classes in relationship to enrollment trends university-wide. In addition, we will present an overview of the characteristics of our online courses and present findings from recent research on online instruction.

Online course delivery has been warmly embraced by many of today's students. This phenomenon, as indicated by numerous recent studies, is not unique to Golden Gate University, but evident at a number of other universities.

Convenience is the most important benefit perceived by online students of the asynchronous online delivery format. Students appreciate the flexibility of course access at any time and in the comfort of their homes or at the office. That online courses are well received by Golden Gate University students is not surprising, as our student body is predominately composed of part time students who work full time.

Three years ago the math faculty was asked to put a number of courses online. Anticipating difficulties, the math faculty worked extremely hard to design courses that somehow transcend these problems. In the initial stages of development we feared that the online popularity would not extend to graphics-based courses such as those typically taught in the Math department. How would the student draw a graph or write a formula over the Internet without first learning sophisticated software packages? How would they communicate with faculty and classmates other without this capability?

In this paper we will present enrollment trends of online math classes versus traditionally delivered classes in relationship to enrollment trends university-wide. In addition, we will present an overview of the characteristics of our online courses and present findings from recent research on online instruction.

Aside from the convenience factor, a number of other facets have contributed to the online success of our math courses.

- Serious attention and effort was paid to the design and utilization of effective online pedagogy. Our content delivery, which can be considered "online lectures", consists of multi-faceted activities. For example, in our Introductory Statistics course, we include PowerPoint slides, worked out examples, Excel projects, "micro lectures" disguised as learning objectives, online quizzes, extra credit essay assignments, etc. We partition the content of courses into weekly modules that contains the same structure week after week so that students know exactly what to expect. We encourage discussions by suggesting interesting topics and provide frequent and positive feedback, where possible.
- The online format is basically a learner-centered format as the student sits alone in front of the computer. The instructor must communicate with the student in both static and dynamic ways. All the above mentioned conference postings, except the quizzes can be considered as static communication from the instructor to the student. Our primary dynamic communication is via asynchronous conferencing and email. As many researcher have found, the student is less intimidated and more willing to "talk" online than in the FTF classroom.
- As many of our students have jobs that require computer and Internet skills, their conferencing and working with each other online is perceived as developing good online working skills.
- The emphasizes our statistic course places on the use of Excel is seen as an advantage by the students as most of them perceive that being facile in Excel is an added advantage at work.

Our online students may feel some awkwardness about the inability to readily use symbols or draw graphs over the Internet. They have found ways to get around this and were able to achieve a deeper understanding of the concepts by trying to express themselves verbally.

Analysis of enrollment trends in multi-section key courses throughout the university reveals that students are replacing the face to face (FTF) traditional course with online instruction. Furthermore, this trend is the most clear and evident for math course. In fact, while total enrollments have decrease throughout the university, the math enrollments have stayed about the same. The decrease in FTF enrollment of math courses has been offset by online course enrollment increase.

# Teaching Mathematics by Means of *MathTrainer*

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**Abstract:** This paper describes *MathTrainer*, a system that guides the student while learning mathematical concepts that involve symbolic manipulation, like the resolution of Ordinary Differential Equations. *MathTrainer* shows students the steps that are required to solve exercises with symbolic calculations. The exercises are posed by the system. The student can also modify some of the formulae that appear in a statement and ask *MathTrainer* how to solve it. *MathTrainer* identifies the type of exercise by means of a pattern matching mechanism and then it shows successively the tasks that have to be accomplished in order to solve the exercise. *MathTrainer* has been integrated in the *MathEdu* teaching environment which includes an authoring tool for the design and interactive resolution of exercises of mathematics. Thanks to this integration, *MathEdu* allows the student to learn first how to solve problems of different types; after this, s/he can practice and get feedback about his/her actions.

## Introduction

During the last years new tools have been developed that simplify the design of generic user interfaces using *Programming by example* techniques (Myers, 2000; Cypher, 1993). In principle, these techniques can be used in an authoring tool in order to make the work of the teacher who designs the contents of a course more intuitive and simpler to accomplish. This kind of environment facilitates the discovery by the teacher of the difficulties the student can have when working by himself, as well as the enhancements that can be incorporated to the course under development. Moreover, the mechanism of generalization allows the teacher to work just on a specific example or problem and later on to extend his work to a whole set of situations that include the original one as a special case. The development of a module with this feature for an authoring tool is very complex, but it is a task that has to be accomplished just once. Once this work is done, it is reused constantly by the different course designers in each course they develop. Moreover, in this way the course designers do not need a deep knowledge of the language or technology that is used behind the scenery, so most teachers can use the tool without any special training.

In this paper we describe how we have enhanced a tool based on *Programming by example* for the interactive resolution of problems of Mathematics, *MathEdu* (Díez 1999). A new module, *MathTrainer*, that guides students through the steps that are required to solve exercises that can be specified dynamically, has been added to it. The exercises solved by *MathTrainer* are posed by the system. The student can also modify some of the formulae that appear in a statement and ask *MathTrainer* how to solve it. *MathTrainer* identifies the type of exercise by means of a pattern matching mechanism and then it shows successively the tasks that have to be accomplished in order to solve the exercise. Thanks to the integration of *MathTrainer*, *MathEdu* allows the student to learn first how to solve problems of different types; after this, s/he can practice and get feedback about his/her actions. In case the student repeatedly makes some mistake while s/he is practicing, *MathTrainer* takes control of the session and shows him/her how to finish the resolution of the problem s/he is trying to solve. *MathCAL* (Mei-Chuen, 1999) does something similar for problems that involve trigonometric calculations. The enhanced *MathEdu* system can be used to develop complete interactive courses about mathematical subjects that

involve symbolic manipulation, like the resolution of Ordinary Differential Equations. Both *MathEdu* and *MathTrainer* are developed over *Mathematica*<sup>®</sup> (Wolfram 1999).

### Brief description of *MathTrainer*

From the point of view of the student, *MathTrainer* is just a program that allows him/her to visualize sequentially a presentation of an example. At the beginning of an explanation the student can choose the type of problem to be explained, and he can also modify the formulae that appear in its statement. For example, when seeing how to solve a linear differential equation, and after having seen a simple example like the resolution of the equation  $y' + y = 1$ , the system will explain that any function of  $x$  and  $y$  that is linear in  $y$  can appear instead of  $y$ , and any function of  $x$  can also appear instead of 1. The student can then ask the system to show him how the solutions of equations like  $y' + y/x = x^2$  can be computed, and *MathTrainer* will then give him/her a step by step explanation of the resolution of this problem. In case the example proposed by the student is not appropriate, the system will tell him/her that, and it will also let him/her know whether his/her example can be already solved using some other method s/he has already studied with *MathEdu* or even if s/he is expected to study problems like the one s/he has posed later on during the course.

Another way in which *MathTrainer* is integrated with the previous *MathEdu* module for interactive problem resolution by the student is by giving him/her explanations about how to go on in case of a repeated mistake while solving a problem, or in case he/she just asks for this help. In the original version of *MathEdu*, this kind of help could be provided only in case the designer of the course had foreseen its necessity, and in this case, the designer had to specify the whole explanation *by hand*. This limitation of *MathEdu* was especially important, since the same situation appeared at every step of the student's work, so the amount of work involved if the designer wanted to give support for the student getting additional explanations at any moment of his work was tremendous.

### About the design process

The design process can be seen as the designer doing *by hand* the actions that both the system and the student are supposed to do at resolution time, and the design engine recording these actions. When *MathTrainer* is showing the student how to solve a problem, either generated by *MathTrainer* itself or posed by the student, it just reproduces the actions specified by the designer and makes by itself the computations that are involved. On the other hand, when the student is solving a problem, *MathEdu* just executes the actions specified by the designer that correspond to the system, and it checks that the expressions introduced by the student through the dialogs are all right according to the corresponding formulae.

*MathEdu* also uses the power of symbolic computation of *Mathematica* to allow the designer to define strategies and cases for each type of problem depending on the specific data associated to the problem. Different strategies for the same type of problem differ by the actions that have to be accomplished in each case, while different cases for the same strategy differ by the pattern that characterizes the values of the initial metavariables that are present in the statement. For example, linear ordinary differential equations correspond to the pattern

$$a\_var' + f\_function[b\_var] a\_var = g\_function[b\_var],$$

that is specified by the designer.

Both when showing the student how to solve a problem and when checking how s/he solves it, the first thing *MathEdu* does once the precise statement has been specified is to determine the possible strategies and cases that correspond to the problem among those specified by the designer. This is done by means of a pattern matching process between the formulae that appear in the statement and the patterns that correspond to each strategy. For example, if the problem consists of solving the differential equation

$$y' + y/x = x^2,$$

then the metavariables  $a$  and  $b$  are associated to the variables  $y$  and  $x$  respectively, and the functions  $f$  and  $g$  are associated to  $1/x$  and  $x^2$  respectively, hence the strategy for resolution of linear differential equations is accepted, which has only one case.

## Conclusions

In this paper we have shown the main features of a highly interactive system to train students in solving problems of Mathematics. The specification of the way the system has to show the student how to solve each type of problem and how it has to check the resolution of each problem by the student is done through the use of a variant of the *Programming by Example* paradigm.

*MathTrainer* incorporates new functionality that was not available in previous tools. This functionality simplifies the definition of courses on subjects that involve symbolic computation. Moreover, these courses allow the student to learn faster how to solve the different types of exercises that they include. The main goal is to make the student feel the system as a collaborator in his learning process.

At this point there is a first prototype of *MathTrainer*. The first version of the enhanced *MathEdu* environment will be ready in some months, and the validation of the system will be done through the development of a course on Ordinary Differential Equations.

Finally, among the future work we have plans to develop mechanisms that allow the system to detect automatically the existence of priorities between different parts of the course, so that in case a student wants to learn some special subject, an optimal path is created automatically that will show him all the necessary prerequisites before teaching him the desired subject.

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## References

Cypher, A. (1993). *Watch what I do. Programming by Demonstration*. Cambridge, MA: The MIT Press.

Diez, F.; Moriyón, R. (1999). *Doing Mathematics with MathEdu*, Proceedings of the IX<sup>th</sup> Conference of Mathematics/Science Education & Technology. AACE. San Antonio (Texas).

Mei-Chuen, J.; Juang, J.; Sun, P. (1999). *An Internet-Based CAL Software for Solving Trigonometric Problems*, Proceedings of the IX<sup>th</sup> Conference of Mathematics/Science Education & Technology. AACE. San Antonio (Texas).

Myers, B., McDaniel, R., and Wolber, D. (2000). *Programming by example: intelligence in demonstrational interfaces*. Communications of the ACM, Vol. 43, No. 3, pp. 82 – 89.

Wolfram, S. (1999): *The Mathematica Book*, 4<sup>th</sup> Edition. Cambridge University Press.

# Beliefs, Experiences, and Reflections that Affect the Development of Techno-Mathematical Knowledge

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**Abstract:** The purpose of this paper is twofold. First, the author discusses her views on teachers' development of techno-mathematical knowledge. Second, the author presents key elements from a course and key course activities and experiences that appear to impact preservice teachers' beliefs about teaching and learning with technology, including those activities that allowed preservice teachers to gain a deeper understanding of mathematics by using technology tools.

## Introduction

Preservice teachers "have little experience with how technology can change the nature and emphasis of the mathematics curriculum" (Gay, 1994, p. 172). They often have varied experiences with using technology, as both personal productivity tools, and as a learning tool in their own mathematics experiences. These varied experiences affect preservice teachers' beliefs about using technology as a teaching and learning tool. The author has designed a course to provide many opportunities for preservice teachers to engage in mathematical explorations with technology and discussions of subsequent pedagogical implications. The preservice teachers reflect on their course experiences, compare these with prior learning experiences, and critically evaluate their own beliefs.

As a teacher/researcher, the author is interested in identifying key course experiences that appear to affect preservice teachers' beliefs about appropriate use of technology as a teaching and learning tool. The teacher/researcher analyzed all written work by the preservice teachers to help answer the following questions:

1. What are preservice teachers' beliefs about using technology for teaching and learning mathematics?
2. How do course experiences affect preservice teachers' beliefs?
3. What course experiences seem to be critical in the development of preservice teachers' techno-mathematical knowledge?

This paper will trace the beliefs, course experiences, and reflections of several preservice teachers throughout the author's course. The analysis will be used to inform future development of the course.

## What is Techno-Mathematical Knowledge?

According to the National Council of Teachers of Mathematics, "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p.11). The use of the word *essential* in this statement has many implications for school mathematics, as well as preservice and inservice mathematics teacher education. Not only are teachers charged with a vision of transforming their teaching and students' learning of mathematics, but teacher educators are challenged with the task of preparing teachers who can utilize technology as an *essential* tool in developing a deep understanding of mathematics, for themselves and for their students. I claim that such teachers must develop *techno-mathematical* knowledge. Such knowledge consists of several intertwined elements:

- Solving mathematics problems and constructing mathematical ideas utilizing technological tools
- Using technological tools to justify mathematical ideas and engage others in meaningful mathematics
- Designing experiences with technology that will allow others to develop techno-mathematical knowledge. This includes designing lessons with appropriate tasks and questions, as well as

- interactive technology templates using tools such as dynamic geometry environments, spreadsheets, and web-based technologies (e.g., java, Shockwave).
- Critically analyzing the value and role of technology to make decisions as to the appropriate use of technological tools in teaching and learning mathematics.

## Course Development

A traditional approach to mathematics teacher education assumes that skills needed in mathematics, pedagogy, and technology are distinct entities. In many mathematics teacher education programs, preservice teachers learn their mathematics content separate from their pedagogy content and separate from basic technological skills. These preservice teachers are then expected to combine these separate skills in a classroom as a mathematics teacher. Some teacher education programs do combine content with pedagogy courses, but still have separate technology courses that are often taken jointly by preservice teachers from a variety of disciplines and grade level concentrations. The message behind this type of teacher preparation is that technology is an add-on “subject” rather than a subject-specific learning tool. Garofalo, Drier, Harper, Timmerman, and Shockley (2000) set forth guidelines advocating integrated and appropriate uses of technology in mathematics teacher preparation.

Teaching and learning mathematics effectively in a technological classroom requires multifaceted knowledge that extends beyond what is typically learned in mathematics methods courses. The work of Garofalo et al (2000) can help teacher educators weave appropriate technology experiences into mathematics methods courses. However, several mathematics teacher education programs (e.g., University of Georgia, North Carolina State University, Pennsylvania State University) have taken another approach -- making a separate course in technology and mathematics education a required part of a secondary program, in addition to existing methods courses. Such courses provide opportunities for preservice teachers to learn how to use technology to better understand mathematics themselves and promote students’ learning of mathematical concepts. Thus, these courses are a blend of mathematics content, pedagogy, and technology.

“Teaching Mathematics with Technology” is purposefully designed to put an emphasis on teaching and learning a variety of school mathematics topics (e.g., Pythagorean theorem, sequence and series, correlation and linear regression, rational numbers). A primary goal in the course is for teachers to deepen their understanding of fundamental topics in secondary mathematics through using technology such as dynamic geometry tools, spreadsheets, interactive websites (e.g., [www.exploremath.com](http://www.exploremath.com)), graphing calculators, and probability simulators. The teachers are engaged in solving rich mathematics tasks with technology, discussing pedagogical implications of using technology in mathematics, reading the NCTM *Principles and Standards* (2000) and various articles describing classroom practices, and reflecting on course experiences and reading.

## Data Collection and Analysis

The one semester course met once a week for 2 hours 45 minutes and included 6-9 hours of assignments each week. During the semester studied, there were 9 students enrolled: 3 Master’s level teachers, 4 undergraduate preservice teachers, and 2 lateral entry teachers who were currently teaching without a license and taking courses towards certification. The targeted students for this paper are the 4 preservice teachers.

As teacher/researcher, I gathered data from a variety of sources for the purposes of assessment as well as research analysis. Each teacher completed a pre- and post-survey to assess their beliefs and attitudes and to document their own comfort level and past experiences with using technology tools. Throughout the course, they completed many assignments in which they used technology to investigate a variety of mathematics tasks. As part of these assignments, the teachers reflected on how the technology helped or hindered their own understanding of the mathematics, as well as how such use could help or hinder their students’ understanding. The assignments were submitted and graded electronically. Thus, there is a chronological electronic trail of all written work, including word processed documents and The Geometer’s Sketchpad and Microsoft Excel files.

I coded the electronic documents and files based on the preservice teachers’ 1) mathematical beliefs, 2) technology beliefs, 3) pedagogy beliefs, 4) developing mathematical understanding, 5) critical analysis of role of technology in developing understanding, and 6) experiences that seem to affect 1-5 above. The evidence supporting each category was further analyzed chronologically to establish patterns of beliefs and how

experiences and reflections may have affected an apparent shift in belief. (I purposefully used *apparent* since there is no evidence of how these preservice teachers put their beliefs into action in a classroom. Such evidence may be gathered as part of a longitudinal study of teachers' development of techno-mathematical knowledge.)

### Shifts in Beliefs During Course

Two students (Nick and Gary) had used graphing calculators extensively since high school and had many experiences using computer software in mathematics and statistics as well as for advanced personal productivity (e.g., web development, programming). The other two students (Nancy and Trina) had limited experience with graphing calculators in math courses and only used computers minimally for personal use (e.g., word processing, email, internet). Overall, the initial beliefs of the 4 preservice teachers indicated they believed technology should be used as a complement to "regular instruction" and feared students' dependency on technology to do math "for them." All of them expressed that technology should be used *after* paper and pencil skills were mastered. They emphasized the quick computational and graphing features (e.g., "plugging and chugging", saving time) of technology and appeared to have little or no experience with how technology could be used to develop a deeper understanding of a mathematics concept.

During the course, each of the preservice teachers experienced several "a-ha" moments while using technology to explore a mathematical idea. Nick and Gary often made advanced connections between school mathematics concepts and their college level mathematics. They also proved to be very good at posing questions or tasks to their classmates during group work to allow others to use technology tools to further their understanding. Nancy and Trina were not as comfortable with the technology and often struggled with the mathematical ideas. However, both of them were tenacious and expressed their enthusiasm and pride when they felt they understood a mathematical idea or were successful in solving an advanced task. All of them gradually showed more evidence of careful thought about how and when technology should and could be used. They still advocated the use of technology as a "time saver" to alleviate computational constraints; however, their focus shifted from skill-driven mathematics towards understanding mathematics concepts and solving real world and higher level problems than they remembered solving themselves in high school.

At the end of the course, Nick, Gary, and Nancy had made evident shifts in their beliefs about how students learn mathematics and the appropriate role of technology in that process, especially in engaging students in developing understanding and making connections with various representations. Trina still held strongly to the belief that that technology was best used after students had learned mathematics in a more traditional manner. Although she often reflected that she felt technology helped her understand mathematics better, those experiences did not appear to affect her core beliefs.

### Tracing Development: A Glimpse at Nick and Nancy

To further illustrate shifts in beliefs, I have included a brief synopsis of Nick's and Nancy's work throughout the semester. Recall that Nick was an advanced user of technology before the semester began, while Nancy only had minimal experience with technology. Nick was also a high achieving student in mathematics while Nancy was an average mathematics student. Nick was a senior taking this course concurrently with a 5-week mathematics methods course (6 credits) and 10 weeks of student teaching (8 credits). Nancy was a junior and had only taken a sophomore level course that was an introduction to mathematics teaching and learning.

#### The Development of Nick

On the pre-survey, Nick expressed his belief that calculators should be disallowed below 5<sup>th</sup> grade, minimally used through Algebra I, and compliment classroom instruction in Geometry – Calculus III. In the second week of the semester, he further noted "I believe in not letting students use technology until they have first demonstrated the ability to reason critically and to think mathematically." Nick appeared to believe that technology could impede a student's ability to think mathematically. I conjecture that this stemmed from his prior experiences with technology being used as a time-saving computational or graphing device, and little experience with teachers asking high level questions and posing challenging tasks with technology tools. However, during the 5th week of the course, Nick did some advanced work in an assignment to use dynamic



visualization tools to analyze the standard ( $ax^2+bx+c=y$ ) and transformational ( $y=a(x-h)^2+k$ ) form of a quadratic function to discover how each parameter affected the shape of the graph. In reflecting on his work he noted:

I had a pretty good idea about how  $a$ ,  $h$ ,  $k$ , and  $c$  affected the graphs of quadratic equations before I began the assignment. But until this, I had never understood how changing  $b$  changes the graph. I was intrigued to see that changing  $b$  will cause the vertex to move in a parabolic path. Likewise, I hadn't realized that changing  $a$  causes the vertex to move along a line. [He further developed the equation for the parabolic path traced by changing  $b$  and the line traced by changing  $a$ . The next week in class, he excitedly led the class in discussing his discovery.]

Nick experienced using technology to gain a deepened understanding of a mathematics concept. I believe this was a critical moment in his development of techno-mathematical knowledge.

During the 6<sup>th</sup> week (Nick's first week of student teaching), we did an investigation of sequence and series using multiple representations. Nick was enthralled during the investigation and continually used the technology (spreadsheet and dynamic geometry) to conjecture, test, and make connections on his own. In reflection of the investigation and subsequent assignment, he wrote:

"I liked especially the sequence generator that we used in Excel ... Excel makes it much easier for students to focus entirely on pattern recognition. In this way, I feel that using the spreadsheet can definitely enhance teacher instruction and/or the learning of sequences and series. I also liked the idea of using the geometric spiral to represent the infinite series. Clearly, this representation helps students to 'organize their thinking.' You can use GSP [The Geometer's Sketchpad] to draw the other three spirals [he did this on his own] ...The geometric spiral will also appeal to so-called visual learners."

In the following week, Nick reported that he had used a geometric spiral to model infinite series in a precalculus course. Although he did not have access to technology in the classroom, he had students construct a spiral on paper and make connections with the numerical and symbolic representations of an infinite series. I feel our investigation with sequence and series helped Nick understand the value of multiple representations and affected his pedagogical approach to teaching infinite series. Not only did he incorporate multiple representations, he claimed students were engaged in building connections while he asked guiding questions.

Concurrently in the 7<sup>th</sup> week, Nick was engaged in an assignment for creating an interactive spreadsheet to explore "nearly golden ratios" (Bradley, 2000) from a generalized form of the Fibonacci sequence. Instead of using the recursive formula  $w_{n+1} = w_n + w_{n-1}$  to generate the sequence, one uses the formula  $w_{n+1} = k*w_n + w_{n-1}$  where  $k$  is any integer. The nearly golden ratios emerge from the limit of the sequence of successive ratios. Nick wrote the following about his work on this assignment:

I think it takes much less effort with the spreadsheet to see that the sequences [of successive ratios] always converge to some real limit. The graph also helps to motivate this idea – regardless of what values the students chooses for  $w(0)$ ,  $w(1)$ , and  $k$ , the sequence [of successive ratios] and the graph will always converge to a real number. I was especially intrigued by negative values of  $k$ : I experimented by looking at the relationship between the sequences when  $w(0)$  and  $w(1)$  were unchanged and  $k$  was changed to  $-k$ . I think this could be a good question to ask students – have them explain what happens and why it happens when the first two variables are left constant and  $k$  is changed to its additive inverse.

Not only did Nick use technology to explore a concept that he had never dealt with before, but he extended his experience to imagining high level tasks that would engage students in mathematical thinking with technology tools. This experience was also a critical step in his development of techno-mathematical knowledge.

Another critical shift occurred in the 9<sup>th</sup> week of the semester. After reading an article about third grade students using a spreadsheet environment to investigate rational number relationships (Drier, 2000), Nick wrote:

Prior to reading the article, I had always felt that technology didn't really belong in elementary schools. The gist of my argument was that technology (calculators) might impede the progress of beginning students by no longer forcing them to think. But what I read in the article was quite the contrary – the technology was the motivating factor for the students' to think mathematically.

Nick further shared his thoughts in a class discussion. He emphasized that technology tools could help students develop mathematical understanding but that "it really depends on how the teacher decides to use the tools." It seems that reading the article and using the same spreadsheet environment to explore several mathematics tasks affected Nick's perception of the usefulness of technology to engage young students in mathematical thinking.

Throughout the remaining part of the course, Nick continued to share his ideas and emphasize the importance of using technology appropriately to help students understand concepts and "think like

mathematicians,” as well as “to help students with tasks that would otherwise be time consuming and tedious, after the student understands how to do them the long way because [technology] allows more to be done in less time without compromising learning.” Although Nick still has a strong belief in doing mathematics “the long way,” he has shifted his belief that technology necessarily impedes a students’ ability to “reason critically and think mathematically” to one a belief that technology, when used appropriately, can help students “think like mathematicians.” Nick now has experiences stretching his own mathematical understanding by using technology and has read and discussed instances of real students’ engaged in mathematical thinking because of the rich tasks posed with technology. It appears that these experiences have affected the shift in his belief.

### The Development of Nancy

Although she had little experience with technology and was a product of traditional mathematics education, Nancy had a very open mind and was interested in learning about how to teach mathematics. Nancy had some difficulty in her college level mathematics courses – difficulty she attributed to her own lack of understanding. During the first week she noted, “so many times I remember learning how to do a problem, but having no idea of what it meant to do that problem.” She further wrote the following reflection after reading the six Principles from NCTM (2000):

When students can learn and see for themselves, they have a much greater chance of remembering and understanding concepts ...it is hard for students to focus on a lecture for a good length of time and using technological visual aids can help redirect student thought processes back into the right direction. Students are also able to learn and draw conclusions for themselves ... Teachers are lucky to have the resources in technology they have now ... As a pre-service teacher I am very nervous about going into education with the knowledge that I will have to use technology ... I am just not very good with computers.

Nancy is openly honest about her feelings. Although she seems to think students should learn and see for themselves, her notion of “technological visual aids” indicates that she may have envisioned students using technology as a display rather than a dynamic interactive tool.

Several weeks later, after an assignment on creating a tessellation, Nancy noted that “by using the geometer’s sketchpad, students will have a chance to come out of their everyday habit of taking notes and watching their teacher teach. Technology is always good for giving the kids a break.” It is evident that although Nancy may think dynamic geometry tools can help students understand better, she believes that a primary benefit is to give students a “break” from “watching” the teacher. Even though she had several “a-ha” moments while solving mathematical tasks with technology during this course, she appears to be strongly influenced by her own educational experiences, and imagines a classroom where students passively take notes while a teacher provides all the “knowledge.”

During the 6<sup>th</sup> week, a critical shift occurred when Nancy reviewed a videotape lesson of students using graphing calculators to examine patterns and quadratic functions. Reflecting on her analysis, she wrote:

I saw so much interaction between the students and I could really see their brains working. I thought back to my own high school experiences and honestly did not remember ever putting so much thought into any kind of mathematical topic because my teachers never challenged me in quite this way. The material I learned was basically straightforward and here, I saw students learning and discovering ideas for themselves.

Nancy needed to see real students engaged in mathematical thinking with technology. Following this assignment, there was a shift in Nancy’s beliefs and attitude towards using technology as a thinking tool and posing tasks that actively engage students in mathematical thought. During the next week, after successfully solving a challenging task with the generalized Fibonacci sequence and discussing ways to engage students in such a task, Nancy commented “I need to break away from my own experiences and branch out my mathematical knowledge.”

Further, in the 9<sup>th</sup> week, Nancy also seemed intrigued by students’ thinking with technology in the Drier (2000) article. She noted “more often than not, teachers ask their students the ‘obvious’ questions, questions that do not require much thought. But, I tend to wonder exactly how beneficial these questions really are. If deeper thought is not called for, then I do not believe that minds are developed to their fullest potential.” Again, Nancy appears to be shifting her belief about mathematics teaching and learning. She seems to understand the value of appropriate questions and tasks. In addition, she continued to develop her mathematical understanding with technology and continually proposed ways to use the tasks and technology from her assignments with students.

Nancy's culminating course project was a well-planned investigation using dynamic geometry and graphing calculators to engage students in developing theorems related to parallel lines. In reflecting on her course project, Nancy's words, although lengthy, tell an amazing story of her recognition of her shift in beliefs.

When I first heard about this project, I was anything but thrilled. I honestly thought that this would be the most tedious thing that I have ever attempted to do. However, I have actually had fun doing this project and through my efforts to produce a meaningful lesson, I really learned just how valuable technology can be in mathematics ... I have always been a little skeptical of this whole "discovery learning" stuff. I do not remember "discovering" anything in high school and therefore wondered why it was necessary for students to figure out ideas for themselves ... I even felt like "discovery learning" was just some sort of new math fad ... This semester has changed my mindset about this idea and this investigation finalized my new attitude. It just makes so much more sense for the kids to learn things themselves because it requires them to think. There is no doubt in my mind that when students actually have to put thought into figuring something out, then their retention rate is much longer and their comprehension is deeper. The technology definitely adds to the discovery process and in the future I plan to look for opportunities where students will benefit from activities in which conjectures are formed. I know that my own recollection of many mathematical ideas is very vague and I believe that this is because I was told everything rather than being forced to think about ideas for myself ... I do not want my students to have this same problem.

Obviously, Nancy's openness and willingness to learn and her experiences, both prior to and during this course, affected her development of techno-mathematical knowledge.

## Summary

The evidence from this first semester of the course indicates that several of the investigations, readings, and assignments impacted the development of these preservice teachers' techno-mathematical knowledge. The reflections required as a part of most assignments also seem to be critical components for the teachers to critically think about their experiences and communicate with me. What was not revealed in this paper were the comments and questions that I wrote back to each student as I read and graded each assignment. I believe these personal communications established a mutually respectful and caring environment that allowed the teachers to take risks in their own learning.

I will continue to develop the course and analyze my subsequent students' beliefs. However, I believe that much more than a single course is needed to develop teachers with techno-mathematical knowledge who can enable students to use technology as an *essential* tool in learning mathematics. Meaningful experiences need to be woven throughout a mathematics education program, and continue throughout a teacher's career.

Author's Note: The details of the author's course can be found at <http://courses.ncsu.edu/ems480>.

## References

- Bradley, S. (2000). Generalized Fibonacci sequences. Mathematics Teacher 93(7), 604-606.
- Drier, H. S. (2000). Investigating mathematics as a community of learners. Teaching Children Mathematics 6(6), 358-363.
- Gay, A. S. (1994). Preparing secondary mathematics teachers. In D.B. Aichele (Ed.), 1994 Yearbook: Professional Development for Teachers of Mathematics (pp. 167-176). Reston, VA: National Council of Teachers of Mathematics.
- Garofalo, J., Drier, H. S., Harper, S., Timmerman, M. A. , & Shockey T. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. Contemporary Issues in Technology and Teacher Education, 1(1), 66-88.
- National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: author.

# **Making geometry on a virtual environment: a proposal of continuous distance education for teachers**

Márcia Campos, Alex Sandro Gomes and Hermínio Borges Neto

Tele-ambiente<sup>1</sup> is an environment composed of a virtual site linked to applications that favor the cooperative work, in which the interaction among participants is mediated by tele-conference resources (image, sound, text and mail) plus an efficient protocol of real-time file and software sharing among the the working group participants (for instance student-student and teacher-student).

The final goal is to develop a distance learning tool for middle-school mathematics teachers in-service professional development in the state of Ceará, Brazil. The course focus on selected topics on geometry, using software as resources for teaching geometry and emphasize problem solving instead of algorithms and proofs. The aim is to develop teachers' conceptual understanding about geometrical topics and at the same time to provide them with tools for planning their classes using internet and educational software.

The present work discusses a pilot experience for implementing this environment. We are currently developing materials for the course. In a previous study conducted in classroom, we observed how teacher interventions could be reduced. In this work we report on findings with a group of pre-service elementary school teachers at the Universidade Federal do Ceará. We first describe the virtual environment called Tele-ambiente.

## **The Tele-ambiente's Structure**

Tele-ambiente is a learning environment composed of a tool, called TELE, and a group of activities that can be used by this tool in real time. We assumed two premises for initially implementing TELE. The first was to adopt patterns established by organisms such as ITU (International Telecommunications Union) and IETF (Internet Engineering Task Forces); the second was to adopt the Internet as the basic infrastructural environment.

TELE was implemented using the ActiveX controls, a Microsoft™ NetMeeting product. The interface was developed using a group of parameters to control the interface objects presented to users. The features of NetMeeting are supported by industry patterns designated by the International Telecommunications Union (ITU), the Internet Engineering Task Forces (IETF) and other normalization agencies. NetMeeting allows ITU H 232 for audio and video-conference, ITU T.120 for the multi-point conference, and IETF LDAP for directory services.

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<sup>1</sup> Tele-ambiente (Developing collaborative and adaptive applications for distance instruction) is a research project approved by ProTeM-CC (it Programs Thematic Multi-institutional in Science of the Computation) of CNPq, Brazil

## **Pedagogical structure of the courses**

Tele-Cabri it is a distance course conducted in Grénoble by IMAG, of the Joseph Fourier university, for hospitalized children. We choose to adapt it for a group of in-service public schools teachers in Fortaleza, Ceará, Brazil. The teachers usually have insufficient content knowledge in mathematics and geometry.

Initially, we have students' machines linked to an internet server which has already properly learning situations stored and scheduled, an virtual tutor agent, interventional objects and historical of the sessions. These items are integrated in the Tele-ambiente environment allowing sharing of Cabri-geometre software, and oral, visual and written communication.

To explain them, begin why commenting the term tutor it is used. Pavel (1997) says that Balacheff prefers to use this term because it differentiates the teaching through TeleCabri from a simple tutorial program tutorial containing all the answers rigidly structured. In a certain way, TeleCabri works as a program tutorial because it possesses an virtual tutor agent that generates problem-solving situations, intervening in students' difficulties through an analysis of didactic engineering accomplished a priori in possible ways to solve the subject, taking in consideration, still, possible mistakes and the student's difficulties.

When identifying the students' difficulty, the virtual tutor has two roads to proceed: it makes an automatic intervention, running over the intervention objects, or it falls back upon the human tutor. The human tutor's importance resides in the fact that not all the students' difficulties can be solved in an automatic way. In the Tele-ambiente database, we set up the virtual tutor, the interventional objects with the situations problems and texts and images with explanatory and illustrative routes.

To structure the course, we will select part of the official curriculum for the geometry teaching demanded for 5a and 8a grades of the Brazilian middle school teaching. We will create the activities with Cabri-géomètreto allow active construction of the geometrical concepts. We also intend to use other available resources in the Internet for not limiting ourselves to one only application. We intend to use with teachers a methodology that stimulates thinking, so that they can learn how to propose problem solving situations that contribute in an active way to constructing students' knowledge. It is also our objective to prepare the teacher to accomplish the analysis of children's answers.

## Helping Elementary Education Majors Brush Up on Mathematical Modeling: Insights from a Field Test of a New Online Learning Prototype

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**Abstract:** This paper describes insights related to the potential use of on-line training modules with elementary education pre-service teachers for enhancing their experience and background in mathematical modeling. These insights resulted from a field test of a new online learning prototype developed within a National Science Foundation Proof-of-Concept project. The insights listed in the paper are further supported by two URL addresses that link to detailed notes, background documents, and various demonstration pieces.

Various studies and national reports have suggested that elementary education majors are often weak within academic content areas, and particularly mathematics and science (see web-site for references). Such studies and reports often suggest the need for remedial content instruction within teacher preparation programs. However, these future teachers may have a difficult time receiving the remedial help they need within a traditional university setting, since as a elementary education major they are often already taking a full load of courses representing numerous disciplines. Thus, the potential of other approaches, such as the use of on-line learning modules, need to be examined for filling this remedial content need of pre-service elementary teachers.

Related to this potential assistance, several online learning strategies are being examined at the University of Nebraska at Omaha with pre-service elementary teachers, including the use of a new online learning module, developed within a National Science Foundation Proof-of-Concept project related to mathematical modeling. The module and other simulations used represent several innovations in online

instruction, such as a node-based database structure for the instructional pieces. The assessment questions and answers, sequence of nodes visited by the student, and duration of time a student visits an instructional node are all recorded. A detailed map of a student's path through the instructional process is also generated to help in the overall analysis of an individual student's learning process. For more information see the extensive URLs at <http://ois.unomaha.edu/aflearn/index.html> and <http://ois.unomaha.edu>.

The investigation of these online learning strategies is just beginning, but some initial insights are already apparent from the results of some initial field tests. These include the following:

- 1) Response patterns reflect that elementary teachers indeed have a wide range of background and experience in mathematics and mathematical modeling.
- 2) The pre-service elementary teachers sampled recognized that online learning might be a useful strategy for their own remedial work in mathematics.
- 3) The pre-service elementary teachers were at first uncomfortable and impatient with the "discovery learning" approaches used within the modules and simulations.
- 4) The pre-service elementary teachers eventually preferred to work in groups of two or three, rather than to work on the on-line activities individually.
- 5) A periodic discussion component (that accessed a real teacher) was seen as essential by the pre-service elementary teachers when using on-line learning activities.
- 6) After some initial resistance with the on-line instruction, the elementary pre-service teachers eventually embraced on-line training that contained some periodic discussion components.

The overall vision for the use of such technology based learning environments is one that is consistent with many national visions and documents. For the pre-service elementary teacher, the use of such learning environments within methods classes may not only help them extend their own backgrounds, but also allow them to experience a potentially powerful new learning environment that will become increasingly available to both themselves and their elementary students.

# **The Modern Classroom: Using Portable Wireless Computer Networking in the Classroom**

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**Abstract:** In the Spring semester of 2000, the College of Education at the University of Illinois sought to alleviate the lack of available class-time in a standard computer lab by implementing a laptop computer lab that could be transported throughout the building and connected to the World Wide Web via a wireless network connection. This paper reports on the usefulness of such a system in a higher education classroom and on the support aspects necessary to implement such a system and control for user needs. Views by our learners from a one-semester course in advanced statistics reveal that a wireless computer lab is effective in providing students with motivation, mobility, and learning while providing the instructor with increased pedagogical choices.

## **Introduction**

Technology continues to be at the forefront of the modern educational system. The latest technology to inundate classrooms around the world is wireless networking. The recent affordability and speed of wireless networks combined with the power and affordability of current laptop computers have allowed portable wireless computer labs to become a reality in many schools. The College of Education at the University of Illinois serves as no exception to this rule. In order to alleviate the overcrowding of a desktop computer lab, a portable solution was implemented that would also take advantage of recent advances in wireless networking. This paper describes this wireless computing environment and the views by our learners from a one-semester course in advanced statistics. Summaries from the survey are shared along with anecdotal comments from our participants. Finally, conclusions are drawn concerning the effectiveness of the system during our trial.

## **Advantages of Portable Wireless Computer Networks**

Wireless computer networks offer schools a big advantage (mobility) over hardwired network technology. Using a wireless network with laptop computers allows students to use computers as they move around a classroom without having to be connected with a network or power cable. With the use of the wireless network on a portable computer comes greater flexibility and mobility for students and classroom activities.



As the wireless network technology continues to improve and the costs decline we will see more and more schools turning to wireless solutions for their local area networks (LANs). Wireless LAN computers (connected to a server through PCMCIA cards that send and receive) provides classroom instructors with greater flexibility in gaining access to the Internet at anytime from anywhere within reasonable distance. It is also a benefit in school communities that are experiencing rapid student population growth where the installation of a wireless LAN is common place in portable classroom settings. Having a wireless LAN allows students and teacher to communicate with each other without cables, sending and receiving data, accessing the Internet and printers.

Recent support systems (LearningStation.com) are beginning to provide enabling, real-time instructional tools through wireless web-based solutions. With these support systems schools are able to gain valuable information in setting up real-time wireless network access and networked computers while providing a more enhanced learning experience. Schools that are using these wireless computing environments are reporting results that are linked to improved performances on statewide achievement tests (math & writing), improved attendance, improved performance on a writing proficiency test, improvement in students' and teachers' research skills, an increase in student motivation and parental involvement (Greaves, 2000).

### **Overview of a Wireless Networked, Portable Computer Lab (iLab)**

The functioning of our iLab is relatively simple. Apple iBook computers with upgrades to give 96 MB of RAM and Apple AirPort wireless networking cards provide the core component. The networking is achieved via radio signals, thus no line of site contact is required for the system to function. The iLab is contained on a cart that provides mobility throughout the College, security of system components, and a custom built recharging system. Once the iLab is in the classroom, students retrieve their assigned laptops. Within a few minutes, the class can be exploring the Internet or performing other computer facilitated activities.

Our iLab implementation took a student centered approach. Enough computers were supplied to provide a one-to-one student to computer ratio when applicable. Furthermore, the wireless aspect is primarily from the student's perspective. The instructor computer was an Apple G3 laptop. This provided an external monitor/projector connection, but the computer was not included on the wireless network for security reasons. Therefore, the instructor's computer still required a network cable. Furthermore, to insure optimal signal strength, the Apple AirPort network hubs were built into the cart that goes into the classroom with the computers. These hubs also have to be hard wired to the College's network and plugged into a power outlet. To allow connection of all devices to a single network jack and power outlet within all classrooms in our College, a switched Ethernet hub and a power strip were also built into the cart. A single network cable and power cable that extend from the back of our transport cart are then connected within the classroom to activate the system. These additional wires are transparent to the students who need only to pick up their laptop to begin working.

### **New Roles For Classroom Teachers and Students**

With the use of wireless technology a new role emerges for the classroom teacher. Greaves (Greaves, 2000) reports that with the use of wireless networks one sees an improvement in collaborative learning and support for more individual, small group and whole class projects. For teachers that use wireless systems, Greaves reports that teachers are using technology for more effective instructional strategies that allow students to discover knowledge and to develop their own personal learning and growth. Many others have concurred. Bradshaw and Massey explain, with laptops, the students learn advanced communication skills and computing techniques in a laptop-enhanced classroom. Also, the PLAIT project (Gardner, 1994), the Copernicus Project (Fouts, 1997), and Project Pulse (McMillan, 2000) among others have all demonstrated the utility of laptops in education. A common theme among the studies is enhanced student centered learning with increased pedagogical choices for the instructor in a laptop-enhanced classroom.

Few projects, however, have examined the impact that wireless networking in conjunction with portable computing can have on the instructor and students. Stetten and Guthrie (Stetten, 1995) report how diffuse infrared wireless networking lends itself well to collaborative, computer-enhanced classroom experiences. They enumerate a number of useful teaching techniques or modes of instruction. Two of their

four modes utilized collaboration while all four modes were enhanced by the laptop presence. The wireless computing provided flexibility that they argue could "serve well for any subject in which human-computer interactions capture the intellectual or creative process."

Such positive classroom environments do not create themselves. Our studies have shown that the positive application of technical components is more than a simple show and tell. Beyond network and computer presence, the instructor needs to have instructionally sensitive programs and activities for use in an iLab environment. Some idea of activities using the technology needs to exist and the instructor needs to have the ability to transform those ideas into functional lessons. When necessary, the instructor must be willing to forgo ingrained power systems within the classroom so that the students can be empowered to carry on their own learning at their own pace within limits that the instructor can still set. In all, the system must provide added value to the educational experience of instructor and students alike.

### **Model for Supporting an Instructor's Use of a Mobile Computer Lab**

Fundamental to the use of a wireless computing environment is the organization of educational materials that can be shared and implemented with a portable classroom environment. To achieve added educational value, some form of support model is necessary to provide the instructor with the tools and knowledge to make use of such materials. In this section we address the support structure that is needed to facilitate a professional learning environment for both the students and the professor. This system must insure computer and networking functionality while providing instructional support for instructors and students so that the computing environment is best utilized to benefit all.

Any use of equipment, independent of its technological nature, requires a support system. An expensive, high-tech piece of equipment quickly becomes an expensive paperweight if the appropriate steps are not taken to ensure the continued functioning of the system as well as instructor support and training. It is only through a continuous stream of communication between the support staff and instructors that viable and effective technology use can be achieved.

Numerous approaches can be executed to maintain a constant connection between the instructors using the iLab and the support staff. To begin with, a communication channel must be opened before the technology is introduced. In our system, all instructors planning to use the iLab must attend an orientation meeting prior to iLab use. Several reasons exist for establishing early communication lines. For one, independent of the desire to implement a system, such action should not be taken if it is against the best interest of the students. It is possible that an alternative method could meet the student's needs better. For example, a video resource may be available that could serve the students as well as World Wide Web access. Secondly, the earlier that communication lines are opened, the more time they have to develop into a functional state. Only once the instructor is comfortable communicating with the support staff can a free flow of information occur to the instructor's benefit. Finally, the support staff must know the needs of the instructor in advance. Early warning is required for the support staff to install required programs, research program effectiveness, test and correct any program conflicts that may exist in the computer systems once new software is installed, and to schedule maintenance of the systems around the class schedule of the instructor.

Once the computer components are in use, the communication between staff and instructor must continue. An instructor needs to feel comfortable with reporting a problem and confident that such problems will be corrected in a timely manner. This comfort leads to greater instructor satisfaction, which can translate into a more diverse and more regular usage of the computer lab in the classroom to the students' benefit. Furthermore, every time that the system is deployed, the support staff should be aware of any changes in instructor needs so that concurrent changes can be made in the computer system to best meet the new needs. Interaction with the instructor was maintained in our system through email, phone, and in-person contact every time that the system was used. The support staff maintains a willingness to communicate and to help the instructor as needs arise.

Helping the instructor is a key concept in creating effective programs. Effective technology use requires more than a simple understanding of how a piece of equipment is turned on, and more too than an understanding of how to start and run a computer program. Instructors should also be given support in the area of ideas. While the instructor is a content expert, the instructor may not be a computer assisted instruction

specialist. Therefore, it becomes an important function of the support staff to complement an instructor's existing strategies with ideas concerning how to integrate technology to best meet the course's needs.

A final step in communication between instructor and staff is evaluation of the systems usage. Post-use meetings were established to evaluate the effectiveness of the system and to search for solutions to any problems that may have arisen. Not all problems may have been reported during a semester, since some may have been small or have slipped the instructor's mind. Also, while some problems may have been small on a single occasion, a continued occurrence of a small problem can become a big problem over time. Therefore, the support staff needs to identify these problems and make corrections prior to future system usage.

Not all requirements to system's needs can be gained directly through communication. Support staff should understand that while an instructor knows the needs of the students, these needs might not be reported if the instructor is unaware that these needs can be met with technology. In other words, the instructor does not always know what is possible. Hence, a support staff needs to be proactive and anticipate student needs. Through anticipation, lessons can be envisioned that make more effective use of the system than first envisioned by the instructor, which can lead to greater satisfaction for the instructor, the students, and the support staff from a job well done.

## Results & Discussion

In addition to analyzing feedback from the instructor and knowledge of support personnel as outlined in discussions above, pre and post surveys were administered to the 26 students attending a graduate statistical methods course in a Midwestern university setting. The pre survey asked seven background questions and nine Likert scaled (Responses ranged from 1 to 5 where a 1 represented Agree and a 5 represented Disagree) items regarding the use of computers and wireless networks. The post survey consisted of 10 Likert scaled (Similar response scale as the pre survey) items and two open ended items focusing on wireless networking. Copies of the survey are available from the website [<http://eval1.crc.uiuc.edu/site2001.html>]. The respondents were represented by 11 males and 15 females with an average of 8.6 years of computer experience, 4.3 years of Internet experience, predominantly owning PCs (85%, with only 1 student not owning a personal computer), and typically spending 1.7 hours a day on Internet. No significant differences existed in average computer experience or knowledge based on student status or gender in our study.

The responses to the open ended items revealed the student's perspective on the use of wireless networks in classroom settings. Overwhelmingly, the responses were positive in nature. Figure 1 highlights some of the responses.

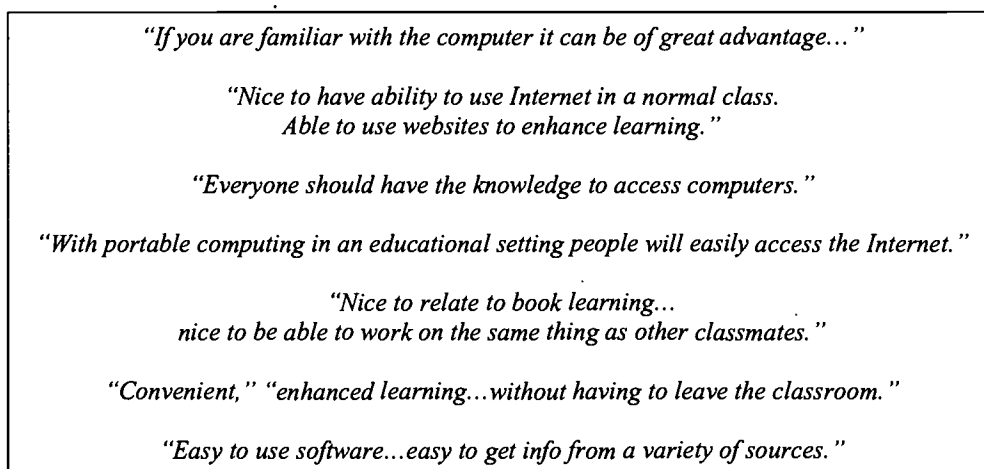


Figure 1: Student Responses to Use of a Wireless Computer Lab

The results from survey items reveal a strong positive level of agreement with the questions concerning the effectiveness of wireless computing. The mean level of agreement to the question on excited about the use of wireless networking" was a mean of 1.62 where 1 represents a high level of agreement.

Similarly we see a high level of agreement ( $M = 1.54$ ) with their response to the question of “using wireless networks in the future.” The majority of the students reported not understanding the security issues involved with wireless networking ( $M = 2.85$ ). From the Post survey we see the students reported high level of agreement ( $M = 1.6$ ) with the “enjoyment” of using iBooks in this class. The use of the iBooks created a greater sense of the security issues as end of course the students rated the question of understanding the security issues involved with wireless networking with a significantly higher level of agreement ( $M = 2.1$ ). Finally, the majority (85%) of the students viewed the use of wireless networking as enhancing their learning experience ( $M = 2.2$ ).

Several enhancements to learning became evident from our study (Figure 2 provides a model for our learning environment that includes student outcomes). First, an iLab enhances student mobility. The computer now goes with the student to where learning is taking place instead of the student going to a computer environment that may inhibit typical classroom learning. This mobility provides increased usability of computer-based and Internet-based learning. A second enhancement is motivation. The iLab places the students in control of their learning as they pace themselves through a choice of computer-based activities. This control, along with a general enthusiasm over computer use, can be a motivating factor for many students.

This control requires that proper planning occur to develop and find appropriate materials prior to lessons. In our class, value was added through simulations of statistical concepts that provided visualization of abstract concepts that the students could manipulate to yield better representations of themes. The computers provided real-time representations of data in a visual format. Student participation seemed increased and students mentioned when questioned that the visualizations provided them with a better understanding of statistical concepts.

Appropriate materials can also merge with student assessment. Through the use of online quizzing, assessment choices for the instructor increased while overall meaningful learning was increased. Online quizzing provides instant feedback to students, thus correcting misconceptions while the thought processes that led to them are still fresh in the student’s mind. In addition, students can complete multiple non-identical quizzes over the same material through “quiz banking” on the computer, thus providing multiple opportunities for reinforcing concepts. Finally, placing the quizzes in an online format provides the student with control over quiz speed, since the quizzes can be taken when the student wants at their own pace.

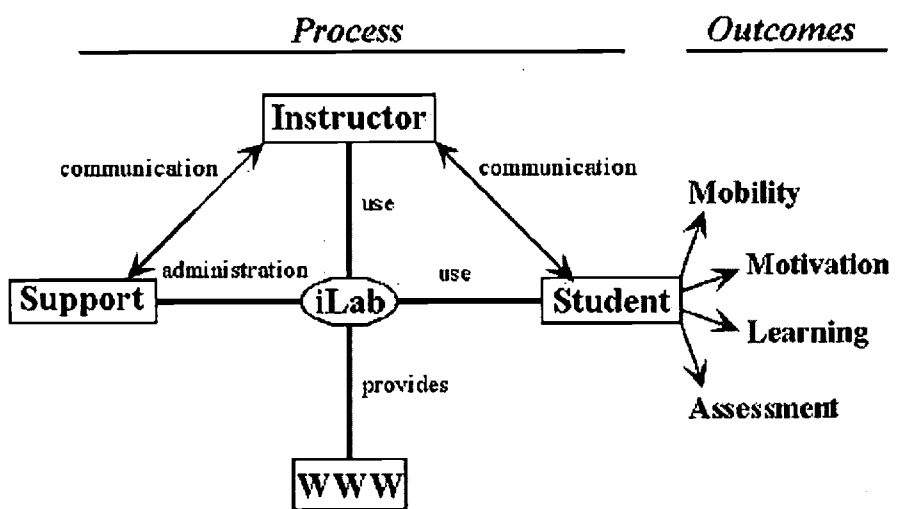


Figure 2 – Simplified Model of iLab Use

## Summary

Through dynamic interaction among computers, support systems, instructor, and students, a wireless networked portable computer lab can provide student motivation, mobility, and learning while providing the instructor with increased pedagogical and assessment choices in a standard classroom context. Use of iBooks

facilitated small group interactions among students pairing up to solve problems, alternating between acting as the problem solver and listener. The iBooks provided natural small group opportunities to clarify and test student's own ideas and perceptions as well as build teamwork and communication skills. Through this process the students gained insights into their own thinking processes and those of others. Furthermore, active engagement of the students via the iBooks in discussions, practicing what they were learning, and applying concepts and ideas to complex performance tasks can yield added value to the educational experience from the instructor and the students' viewpoints.

## References

- Fouts, J. & Stuen, C. (1997). Copernicus Project: Learning with Laptops: Year 1 Evaluation Report. (ERIC, ED 416 847).
- Gardner, J; and others. (1994). Learning with Portable Computers. *Computers and Education*. 22 (1-2), 161-171.
- Greaves, T. (2000). One-to-one computing tools for life. *T.H.E. Journal*. 27 (6), 54-56.
- McMillan, K. & Honey, M. (1993). Year One of Project Pulse: Pupils Using Laptops in Science and English. A Final Report. Technical Report No. 26. (ERIC, ED 358 822).
- Stetten, G. & Guthrie, S. (1995). Wireless Infrared Networking in the Duke Paperless Classroom. *T.H.E. Journal*. 23 (3), 87-90.

# Integrating Mathematics Education Technologies into Teacher Education at the University of Colorado-Boulder

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**Abstract:** Technology is transforming the face of mathematics education. The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics (Standards)* states "the existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it" (NCTM, 2000). Furthermore, NCTM describes that "the effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used well or poorly" (NCTM, 2000). Evidence suggests that teachers frequently have difficulty aligning their practice with these *Standards* and that they need financial, cognitive and technical support in order to do so (OTA, 1995). This paper describes ways that the University of Colorado-Boulder has infused technology into their teacher education program in general, and their mathematics education program, in particular, in order to "prepare tomorrow's teachers today" to effectively teach with mathematics education technology.

## Teacher Education Technology Framework

The teacher education experience embodied at the University of Colorado-Boulder (CU) is designed to instill in preservice teachers an informed and critical perspective on the uses of new media so that these teachers are provided with the skills and confidence necessary to implement their professional objectives and decisions. The teacher education program at the School of Education at CU has long been committed to training exceptionally well-prepared teachers so that they think critically about education and theories of learning, and establish inclusive classroom environments that foster learning opportunities for all students. Our program is imbued from the ground up with philosophies of teaching toward democratic ideals, and the goals of providing students with constructive, learner-centered classrooms founded on scientific research on education. As we enhance our own capabilities to use – and teach about – educational technology in K-12 education, we are similarly committed to an integrative and systematic approach to teacher education reform. Thus graduates from this new program aimed at preparing teachers to effectively use technology will learn to operate with an inquiring disposition that considers the When?, Why?, and How? of using technology to foster basic skills, knowledge, and conceptual understanding in all students. Because we as teacher educators are obligated to provide our students (preservice teachers) with the best and most up-to-date professional program, we believe that future teachers should be learning to use, teach with, and teach students to think with modern technologies as an integrated component to their teaching practices. Toward this end, educational technology use needs to be infused into the curriculum by education faculty who are modeling technology-proficient instruction, particularly in those courses where preservice teachers develop the subject area expertise they will use in the classroom. In this way, our teacher preparation programs can be sure that future teachers master new instructional strategies, multiple learning styles, and content applications that enable them to make full use of modern technologies for improved learning and achievement.

## Secondary Mathematics Methods Technology Framework

Secondary Mathematics Methods (Methods) is taken by preservice teachers the final semester of their teacher education program, just before they begin their student teaching experience. By this time in their teacher education program students have learned much about the discipline of mathematics, taken education courses examining theories of schooling, teaching, and learning; and have observed and participated in various educational settings. An explicit goal of Methods is to help preservice teachers develop their mathematics teaching philosophies and practice (Peressini et. al., 1998). This course strives to help preservice teachers develop the skill of being able to make connections between the discipline of mathematics and effective ways to help students understand mathematics; the ability to help students to learn their way around the discipline while attending to conceptual barriers along the way (NRC, 1999).

During this course, preservice teachers engage in authentic mathematical tasks. The preservice teacher's participation in these mathematical activities is done in a two-fold manner. First, these aspiring teachers participate as learners in order to experience the reform-based mathematics they will be asked to teach, and then reflect as pedagogues on the instruction and curriculum they have just experienced. Much of the reform-based mathematics that occurs in Secondary Mathematics Methods includes technology rich instruction and curriculum. This technology is integrated through a see once, try once, do once model. In this way, preservice teachers can observe, participate, and reflect upon the Why, When, and How of technology use in mathematics education. Examples follow in the next section.

## Utilized Mathematics Education Technologies

"In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics (NCTM, 2000)." The first author of this paper has identified, and worked extensively with four mathematics education technologies, which have demonstrated particular promise towards enhancing student learning. These technologies fit well with multiple teaching and learning styles, especially reform-based pedagogy, and help to foster connections between students' mathematical understandings and other parts of their in-school and out-of-school experiences. These technologies are Geometer's Sketchpad (KCP, 1997), Graphing Calculator's including Calculator-Based Laboratories, Fathom (KCP, 1999), and The Jasper Woodbury Adventure Series (CTGV, 1992). These mathematics education learning tools can be used dynamically, and when utilized effectively, help to meet NCTM's recommendation of using technology to "blur some of the artificial separations among topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics" (NCTM, 2000).

These technologies afford opportunities to link mathematics curriculum and instruction across subjects and to students' lives (Hovermill, J. & Anderson, K., in progress). Calculator-based laboratories allow students to collect data about real-world phenomena via the use of tools such as motion detectors, pH probes, light sensors, and temperature probes. Students can investigate the world around them by using this equipment and then use graphing calculators or Fathom software to analyze the data they have collected. Fathom is a particularly powerful mathematical exploratory tool since students can simultaneously investigate multiple representations of the data they have collected. While students try to understand interesting data sets they are motivated to learn and apply many mathematical concepts (Finzer, 1999).

The Jasper Woodbury Adventure Series is a video-based learning environment that fosters student problem posing and solving. Fifteen to twenty minute videos are presented to students in Jasper Adventures. Embedded in these videos are critical ideas that are necessary towards solving the multi-step problems presented to the students at the end of the adventures. The developers of the Jasper Woodbury Series comment how their design "anchors" students' subsequent project-based learning (CGTV, 1992). Multiple classrooms investigating Jasper problems are often networked together with other schools and learning communities or "collaboratories" are thus created. Learning communities increase student motivation, provide opportunities for formative feedback and assist towards promoting learning with understanding (NRC, 1999).

Geometer's Sketchpad is another dynamic learning environment, which can be used to help connect aspects of the subject of mathematics, within or outside the discipline. Specifically, students can quickly produce dynamic sketches that can lead to conjectures, investigation, and communication (Schattschneider & King, 1997.)

## Results

Preservice teachers have responded favorably to opportunities to experience technology-based mathematics as learners and as teachers. They have reflected upon these experiences in regards to student motivation and understanding as well as instructional and curricular issues. They have engaged in meaningful dialogue inside of class and outside, via electronic discussions. These preservice have themselves integrated technology into their student teaching fieldwork experiences. Three themes have repeatedly surfaced as aspects of technology-based mathematics. First, these preservice teachers have concluded that technology affords opportunities for rich mathematical investigation. Second, these investigations involving technology can help to make connections within the discipline of mathematics and between mathematics and other disciplines. Third, technology-based investigations can drive instruction and curriculum in authentic, collaborative ways that help to foster learning for understanding. Examples of these preservice teachers' conclusions are below.

Responding to technology-based activity learning in general, one student responded:

I think technology-based activity learning offers a richer way (for students) to understand academic subjects. I think that it provides a contextual framework for mathematics so that students understand connections between mathematics and other disciplines.

Another student responding specifically to a light sensor Calculator Based Laboratory activity, which investigated the fluctuations of fluorescent lights and modeled this fluctuation on the graphing calculator with an absolute value sine function, said:

It is pretty obvious that activities such as the light activity can be interesting, fun, and educational as well. The activity made good connections, it involved thinking about scientific concepts, and also required that mathematical concepts and computations be involved. Activities like this make the learning more fun, simple as that.

Yet another student added:

I think technology-based activity learning helps students ask why formulas are the way they are. They can see algorithms they have learned in class put to practical uses. If they don't have all the content knowledge while working on the lesson, it will force them to ask more questions about the lesson. When we were working on the lesson, because we didn't all know everything, the lesson was more of a discovery lesson. It integrates technology, content knowledge, and group work and allows students to discover answers instead of just being told them.

Another student concluded:

students are able to see how ideas and concepts are interrelated across disciplines. This inter-relatedness makes for a stronger foundation of knowledge for students. I believe it lets them see that content areas do not exist solely by themselves.

## Conclusions

“Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning” (NCTM, 2000). Although NCTM and other educational organizations recommend integrating technology into mathematics curriculum and instruction, this is still



not often accomplished in many classrooms (NRC, 1999). At the University of Colorado-Boulder, we have infused technology throughout our teacher education program and provide students with many opportunities to experiences and discuss the Why?, When?, and How? of technology integration. These multiple learning situations allow preservice mathematics teachers to critically examine issues of teaching and learning and to integrate technology into their personal educational ideologies. Teacher education at CU provides a framework that helps to "prepare tomorrow's teachers today" to teach effectively teach with technology. We utilize a model of observe once, practice once, lead once in supporting preservice teachers towards discovering the affordances for student learning that technology offers and integrating meaningful technology based investigations into their own mathematics curriculum and instruction.

Students in this program have articulated three findings about technology-based activity learning. First, these preservice teachers have concluded that technology affords opportunities for rich mathematical investigation. Second, these investigations involving technology can help to make connections within the discipline of mathematics and between mathematics and other disciplines. Third, technology-based investigations can drive instruction and curriculum in authentic, collaborative ways that help to foster learning for understanding. With continued practice and support these beginning teachers will become the leaders for change in their new schools and help to integrate mathematics education technologies into school mathematics.

## References

- Bransford, J., Brown, A., & Cocking, R. (1999). *How people learn: brain, mind, experience, and school*. Committee on Developments in the Science of Learning, Commission on Behavioral and Social Sciences and Education, National Research Council. Washington, DC: National Academy Press.
- Cognition and Technology Group at Vanderbilt (1992). The Jasper Series: A Design Experiment in Complex, Mathematical Problem Solving. *Design Experiments: Integrating Technologies into Schools*. Hawkins & Collins (Eds.) New York: Cambridge University Press
- Finzer, B. (1999). *Design of Fathom, a Dynamic Statistics Environment for the Teaching of Mathematics*. Key Curriculum Press.
- Jackaw, N. (1997). *The Geometer's Sketchpad*. Key Curriculum Press.
- Schattschneider, D. and King, J. (1997). *Geometry Turned On!: Dynamic Software in Learning, Teaching, and Research*. Washington D.C.: The Mathematical Association of America.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards of School Mathematics*. Reston, VA.
- Peressini, D., Elliott, R., & Knuth, E. (1998). *Lessons learned from the mathematics teacher mentoring project*. Paper presented at the annual research pre-session of the National Council of Teachers of Mathematics, Washington, DC.
- U.S. Congress, Office of Technology Assessment. (1995). *Teachers & Technology: Making the Connection, OTA-HER-616 (Washington DC: U.S. Government Printing Office)*.

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# Teaching Technology With Technology: A Case Study. How One Faculty Member is Integrating Technology Into His Pre-Service Mathematics Methods Classes

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**Abstract:** This case study tracks a secondary mathematics pre-service class. The professor's thoughts give insight into why he is incorporating technology into his curriculum. The pre-service mathematics class is also participating in a pilot project in which wireless Macintosh® iBook computers are put to use. Classroom observations, interviews and documentation of student feedback were used for analysis. This professor's philosophy is that technology should be an instructional tool, a contribution to productivity. This faculty member is pleased with the results of technologies utilized in the course. Students reveal they benefits of technologies used in this methods course. The interesting concern of students was not the use of technology, but "which brand" would offer the best results.

## Introduction

We are all familiar with the critical cries that teachers need better preparation in using instructional technology. Accordingly, we hear that university faculty themselves need a better understanding of technology. When a faculty person does have a high level of technological understanding, is there necessarily a gain in that person's productivity?

This study is motivated in part by economist Robert Solow's often quoted remark "we see computers everywhere but in the productivity statistics," This has often been used to describe the phenomenon called the "productivity paradox." The paradox questions the true benefit of computing technology in the workplace; in simple terms, do we get less done using computers than we used to? Until recently, evidence suggested that the productivity paradox did, in fact, exist. The latest results from the business world suggest that computers are contributing to productivity, at least in sectors of the economy that are themselves highly focused on technology.

Is it possible, that a technology paradox may exist in the process of teacher preparation? To explore this possibility, the author has undertaken a case study of a professor of mathematics education who is highly skilled in the use of computing technology. In this paper, the author presents background information on the subject, and describes the methodological approach in process.

## The Study

For this on-going case study, classroom observations, instructor interviews, and student written feedback were analyzed. Several students were also interviewed to gain their perception of technology, the use of technology in the classroom, and how effectively technology was being used in this particular class. Data from student journals and observations from other faculty augment the study. The themes, which emerge, were examined in the light of the "productivity paradox" and the increasing mandate for educational colleges to produce technologically literate teachers.

Students in this course were also included in a college pilot project, which allowed students access to a Macintosh® iBook computer during this class period. The iBooks computers were equipped with wireless technology, which allowed the instructor and each student access to the Internet anywhere in the classroom. Mathematica®, Geometer's Sketch Pad®, access to e-mail, and Internet browser tools were installed on each computer. Advanced calculators and other technology tools were utilized during some portion of this course.

A portion of this pre-service methods course involves students going out into the schools. The purpose of these school experiences is to observe, help teach classes, and practice many skills, which have been learned during their pre-service instruction. This study did not track the students into the schools during these practicum exercises.

It should also be noted that this professor has a mathematics and a technology background. Prior to his current tenure as a mathematics professor, he was a faculty member in the technology division of this college.

## Findings

The instructor is pleased with the way technology is being integrated into his pre-service mathematics course. Observations and interviews reveal this professor has prepared the coursework and incorporated technology to augment elements being presented. Students have reacted to this form of instruction in a favorable manner. One male student gave the following written feedback. "I thought Wednesday would be a bore ... after all I am an experienced user. Boy was I wrong ... I learned lots of neat features that would work beautifully in my Calc reci as well as the prealgebra we will be teaching." A female student observed, "I enjoy a ... teacher who is trying innovative things in the classroom." Perhaps one male student best summarized the issue of technology in the classroom. "It is amazing what technology will allow you to do in a classroom. It was also amusing ... to see some of the difficulties technology presents and the troubleshooting that is sometimes required."

The use of Macintosh® iBooks created new experiences for both the instructor and the students. Most students found this pilot study very useful and were excited to be part of this project. Students liked being able to work Mathematica® problems in class, so information could be shared and solutions discussed. Students appreciated the fact they could explore web sites as information was being communicated.

The Macintosh® iBook project also presented the instructor and students with several shortcomings. The instructor discussed missed class time when students were checking out machines and returning them to the designated carts. Several students commented that having a laptop in front of them the entire class period lead to distractions, such as reading your e-mail, "surfing" the net, or using other software. Others said, if you are going to be distracted, don't blame it on the computer. Yet, the overall perspective of the iBook project is viewed as a valid and positive experience. Surprisingly, one of the most controversial points of this project arose when class discussion focused on whether Macintosh or Windows computers should be used.

## Discussion

Faculties are divided about "how" and "to what extent" technology should be used in today's curriculums. This Secondary Mathematics professor, at a large midwestern university, considers technology an integral part of his curriculum. Students tend to agree with this perspective. A valuable follow-up to this study would be to track these students into future classrooms and ascertain if technology is being infused into their curriculums. The size of a school and a school district's perception of technology have been factors that can alter a teacher's use of technology.

## References

- Creswell, J. W. (1994). *Research design: Qualitative and quantitative approaches*. Thousand Oaks, CA: Sage.
- Drazdowski, T. A., Holodick, N. A., & Scrappaticci, F. T. (1998). Infusing technology into a teacher program; three different perspective. *Journal of Technology and Teacher Education*, 6, 141-149.
- Medcalf-Davenport, N. A. (1998, November). *Historical and current attitudes toward and uses of educational technology: A work in progress*. Paper presented at third annual WebNet 98 World Conference of the WWW, Internet and Intranet Proceedings, Orlando, Florida.
- Moody, F. (1999). *The Productivity Puzzle: Tech, Ready When You Are*. ABCNEWS.com. From the World Wide Web: <http://www.abcnews.go.com/sections/tech/FredMoody/moody990818.html>

# Museums Meeting Schools: Online and Right On the Mark

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**Abstract:** Schools, individual teachers, and students are entering museums through the World Wide Web instead of the entrance off Main Street. Museum directors are faced with the challenge of designing and using Internet based technologies to enhance the bond between students, schools and museums. This article discusses some of the technologies that museums are using based on an analysis of the Detroit Institute of Arts Education Department's Internet site and like efforts around the world. Each discussion is followed by an example of a museum that is using a particular technology and information directing the reader to a web site with more information. Design considerations for museum based educational web sites are also briefly discussed.

## Introduction

Schools, individual teachers, and students are entering museums through the World Wide Web (WWW) instead of the entrance off Main Street. Museum directors are faced with the challenge of designing and using Internet based technologies to enhance the bond between students, schools and museums. This article discusses some of the technologies that museums are using based on an analysis of the Detroit Institute of Arts Education Department's Internet site and like efforts around the world. Each discussion is followed by an example of a museum that is using a particular technology and information directing the reader to a web site with more information. Design considerations for museum based educational web sites are also briefly discussed.

The following is a description of technologies subsumed under the Internet and examples of their educational use by museums.

## On-line Design Considerations

Designing online educational outreach is not difficult. A few design considerations can greatly enhance the value of a museum's educational web site. The following design tips address some of the technologies more commonly used by museums today.

When designing an educational component of a museum web site, the museum's home page should have a hyperlink that is clearly labeled Education. From there, categories like lesson plans, e-mail, chat, video files, and sound files should be clearly visible.

Web sites should be easy to navigate. A visitor should be able to move between major parts of the web site with no more than two clicks of a mouse. Museums should avoid navigation features that frustrate web surfers like automatic pop-up windows and restricted access to going back. E-mail links should include the name of the institution or individual receiving the mail as well as the address.

Teachers and students should be encouraged to download pictures, graphics, sounds, and video. The site should clearly encourage the downloading of these elements and ensure that they are copyright free and identified as such. Cooperating teachers, students, and museum staff should create many, if not all, of the downloadable elements to avoid the unintentional use of protected materials.

If uploading documents is encouraged, simple procedures for doing so should be clearly visible and easy to find. Uploaded materials should be sent to the museum's web master for review and placement on the site. Since any materials on the site will reflect on the museum, careful screening is important.

Chat events should be clearly announced weeks in advance. Rules for participation should also be easy to find. Use the speaker's previous speaking engagements as a guide for estimating the number of people who would be interested in participating in a live discussion. Make sure that the software and hardware can handle the expected number of chat participants.

Video is a powerful tool on the Internet but usually requires more than a standard system to be received. Plug-ins or special programs for running video should be available through links to vendors who are willing to supply free versions of their software. Avoid video formats that are uncommon.

Directors need to remember that the Internet is not a static environment. Materials, listings, and links must be constantly updated and maintained. The Internet offers museums and other academic communities the most dynamic opportunity available today for collaboration.

It is also important to promote the educational component of a museum's web site. Search engines like Yahoo and Excite make it relatively easy to list a web site in their database. Key words should include terms like lesson plans, teacher resources, and educational activities. Content specific terms like Egypt, mummies, and pyramids should also be included.

### **Examples of Technologies used by Major Museums**

Web pages are the most commonly recognized component of the Internet. Created using Hypertext Markup Language (HTML) and a variety of other programming languages, the vast number of web pages linked together through phone lines and satellite make up the WWW. Most museums already use web pages to post schedules and promote exhibits. They can also be used to post educational materials related to exhibits and events at the Museum. The Detroit Institute of the Arts has created a site off of their home page entitled "*Ancient Egypt: Lesson Plans for Teachers* (Ancient Egypt: Lesson Plans for Teachers, 2000, November 28)." This site includes a selection of cross-curricular lesson plans created by teachers in the Detroit Metro area.

This site is an excellent example of the most desirable relationship between museums and schools. The site was not created simply for teachers but with teachers. This integration helps the museum to offer instructionally sound courseware while giving teachers access to the Museum's extensive collection of artifacts and knowledge.

The lesson plans in the site are tied to the Michigan's Content Standards and Draft Benchmarks (Content Standards and Working Draft Benchmarks, 2000, November 28). These standards are not a state mandated curriculum but they are widely used throughout Michigan as a curriculum guide by school districts. This enhances their value to other teachers who are also teaching a curriculum based on the standards. The site includes lesson plans for Art, Science, Math, Social Studies, and Language Arts. The lesson plans at the DIA site include activities, assessments, goals & objectives, pictures of students and projects, preparations & resources, and comments from the teacher who wrote the lesson plan (Ancient Egypt: Lesson Plans for Teachers, 2000, November 28). In fact, more and more museums are providing lesson plans as part of their educational outreach (Classroom Ready Lessons and Activities, 2000, November 28; Kids Project, 2000, November 28; Online Museum Educators, 2000, November 28; Teacher Resources, November 28a; Teacher Resources, November 28b; Teacher Resource Center, 2000, November 28).

The art of writing a letter is making something of a comeback with the popularity of e-mail. This technology gives students a chance to compose thoughtful questions that go beyond what their teacher, their textbook, and even the WWW can answer. Museums, with their in-house collection of experts can provide many of those answers.

The Milwaukee Art Museum has an e-mail link that allows students to pose questions to some of the artists whose work make up the institution's collection (Ask the Artist, 2000, November 28). Even minor celebrities seem remote and impossible to contact. Teachers dream of giving their students the opportunity to meet and interview the author of a children's story, the successful explorer, and the commander of the latest shuttle mission. Museums can host such interviews through the use of chat technology.

Chat refers to asynchronous conversations over the Internet. Typically, a chat room looks like a small box with a list of names. In the case of a museum interview, the names of the children and the name of the

expert would appear. Students type in their questions or comments, which then appear on the screen in the order a central server receives them. The expert then replies in similar fashion.

The National Air and Space Museum in Washington, DC maintains a link to NASA's Space Team Online Project (Chats, 2000, November 28). This link provides schools with an access to live interactions with various NASA personnel including astronauts. Schools must register to take part in live chats.

Museums frequently sponsor events and lectures. These can be video taped and then converted to streaming video. Most people are aware that video files take up a substantial amount of room on a computer's hard drive. It also can take many hours to download even a small amount of video through a standard modem connection. Streaming video is a technology that sends only a small amount of video through the line at a time while the video is being viewed. The result is substantially quicker access to video on the web and elimination of disk space problems.

Museums like the Tech Museum of Innovation at have begun offering some of their education-related content in the form of streaming video (Online Events, 2000, November 28). Teachers and students can watch lectures and interviews at any time.

Web based video works by connecting a small camera to a web page. The result is live video available on the WWW. Museums with dynamic or active exhibits can display all of the motion using this technology. Bishop Museum and The State Museum of Natural and Cultural History in Honolulu, Hawaii now offer live video feeds of some of their exhibits (Bishop Museum-Quickcam, 1997).

Interactive video is a similar technology. It doesn't display its video images on the WWW but sends them through phone lines in a similar manner. Interactive video using ISDN lines currently offers better quality sound and picture than web-based video. Unlike web based video, which may be available 24 hours a day, interactive video is used for events like lectures and presentations. The Philadelphia Museum of Art offers a program called ArtLine (The ArtLine Distance Learning Program, 2000). Artline is a distance learning initiative used to offer virtual tours of the Museum or interactive events with speakers.

## Summary

Museums can enhance educational outreach through the use of on-line resources for schools, teachers, and students. But to be effective museum directors must understand fundamental design considerations. More importantly, they must reach out to schools, teachers, and students to partner in the creation of educational materials and on-line interactive environments that make the most of museum resources and are relevant to learning situations. Many museums are experimenting with new technologies for educational outreach. With cooperation from other academic communities, museums can find new ways to serve their local communities and the world.

## References

Ancient Egypt: Lesson Plans for Teachers (2000, November 28). Detroit, MI: Detroit Institute of the Arts. Retrieved November 28, 2000, from the World Wide Web: <http://www.dia.org/education/egypt-teachers/index.html>

Ask the Artist. (2000, November 28). Milwaukee, WI: Milwaukee Art Museum. Retrieved November 28, 2000, from the World Wide Web: [http://www.mam.org/html/ask\\_the\\_artist.htm](http://www.mam.org/html/ask_the_artist.htm)

Bishop Museum-Quickcam. (1997). Honolulu, HI: Bishop Museum. Retrieved November 28, 2000, from the World Wide Web: <http://www.bishop.hawaii.org/bishop/quickcam>

Chats. (2000, November 28). Washington, DC: The National Air and Space Museum Retrieved November 28, 2000, from the World Wide Web: <http://quest.arc.nasa.gov/space/chats/>.

Classroom Ready Lessons and Activities (2000, November 28). Washington, DC: Smithsonian Institution. Retrieved November 28, 2000, from the World Wide Web: <http://educate.si.edu/resources/lessons/lessons.html>

Content standards and working draft benchmarks. (2000, November 28). Lansing, MI: Michigan Department of Education. Retrieved November 28, 2000 from World Wide Web: <http://cdp.mde.state.mi.us/MCF/ContentStandards/>

Kids Projects (2000, November 28). Cleveland, OH: Cleveland Museum of Art. Retrieved November 28, 2000, from the World Wide Web: <http://www.clemusart.com/educatn/kidsprojects/index.html>

Online Events. (2000, November 28). San Jose, CA: TheTech Museum of Innovation. Retrieved November 28, 2000, from the World Wide Web: [http://www.thetech.org/exhibits\\_events/online/](http://www.thetech.org/exhibits_events/online/)

Online Museum Educators (2000, November 28). Philadelphia, PA: The Franklin Institute Science Museum Retrieved November 28, 2000, from the World Wide Web: <http://sln.fi.edu/qa98/wiredindex.html>

Teacher Resource Center (2000, November 28).Seattle, WA: Seattle Art Museum. Retrieved November 28, 2000, from the World Wide Web: <http://www.seattleartmuseum.org/trc/default.htm>

Teacher Resources (1998). The Ohio Historical Society. Columbus, OH <http://www.ohiohistory.org/resource/teachers/>

Teacher Resources. (2000, November 28a). St. Louis, MO: St. Louis Science Center. Retrieved November 28, 2000, from the World Wide Web: <http://www.thetech.org/people/teachers/resources/activities/>

Teacher Resources. (2000, November 28b). San Jose, CA: TheTech Museum of Innovation. Retrieved November 28, 2000, from the World Wide Web: <http://www.thetech.org/people/teachers/resources/activities/>

The ArtLine Distance Learning Program (2000). Philadelphia, PA: The Philadelphia Museum of Art. Retrieved November 28, 2000, from the World Wide Web: <http://www.philamuseum.org/education/distance.shtml>

# Bridging Transformational Geometry and Matrix Algebra with a Spreadsheet-Based Tool Kit

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**Abstract:** This report presents technology-enhanced activities designed for secondary mathematics teachers within the context of matrix algebra, transformational geometry, and fractals. It concerns the ways in which a spreadsheet can be used as a tool kit that enables one, through encoding and manipulating matrices, to create fractals with reliance upon *the Chaos Game*. The authors argue that such use of a spreadsheet can help prospective teachers link properties of matrices and concepts of transformational geometry in a meaningful and representational manner.

One of the central tenets of the current reform movement in mathematics teacher education is the appropriate use of the tools of technology in teaching and learning mathematics at all grade levels. The recently published document *Principles and Standards for School Mathematics*, by the National Council of Teachers of Mathematics (NCTM), included technology as one of its six Principles. The technology principle is based, in part, on the notion that computer-enabled pedagogy extends the range of problems that students can access and provides, with relative ease, more representational forms than a pencil-and-paper environment (NCTM 2000).

Indeed, a variety of representational forms relevant to secondary school mathematics instruction can be created through the use of computer graphics, electronic tables, symbolic manipulators, and dynamic geometry. All these types of software have proven to be useful cognitive tools in technology-rich classrooms and have become an important cultural component of the modern educational system. The various technology tools should not be considered in isolation; rather they should be viewed as a tool kit conducive to mediate mathematical action.

Any technology tool kit is a product of social evolution and cultural development. Thus, it may be helpful to structure the discussion on the mediation (through use of technology tools) of students' mathematical action in contemporary classrooms from the socio-cultural perspective (Wertsch 1991). This theoretical perspective distinguishes three major positions associated with a tool kit approach to mediated action: heterogeneity as genetic hierarchy, heterogeneity despite genetic hierarchy, and non-genetic heterogeneity. The first two positions admit an inherent ranking of available notation systems (electronic tables, graphing tools, etc.) within a learning environment. The third position does not admit the existence of the hierarchy of a cognitive effectiveness among the elements of a tool kit. In this report, we adopt the third position. More specifically, non-genetic heterogeneity of a mediational tool kit implies that there is no inherent ranking within the manifold of representational formats in human mental functioning. As far as a representation of complex ideas in a computer environment is concerned, this position is in agreement with Kaput's (1992) claim that each notation system in a technology-rich tool kit reveals more clearly than another some aspects of a mathematical concept and hides some other aspects of the concept. From this position, a tool kit approach to mediated action brings about the abundant property of a representational variety in which the whole exceeds the sum of its parts. This paper will show how a variety of representational formats in the context of transformational geometry and matrix algebra can be examined from the third position.

The metaphor of a tool kit in the context of technology-enabled mathematics instruction means an array of representational formats that mediate students' mathematical thinking in a technology-rich environment. The major claim of a tool kit approach to teaching and learning mathematics in a computer



environment is that the variety of qualitatively different representational formats (notation systems) provided by the environment affects students' acquisition of new concepts in different ways. While there is basically no inherent ranking among different types of software used in secondary mathematics classroom, the appropriateness of a particular type for a specific classroom depends on the topic being studied. Therefore, the metaphor of a tool kit in the context of a technology-rich mathematics classroom may be associated with the non-genetic heterogeneity position.

Variety in representational format may seem to require diversity in software, whereas many classrooms lack diverse software tools. However, the non-genetic heterogeneity position appears to be particularly helpful in supporting an alternative idea of using a single computer application - an electronic spreadsheet - as a tool kit (Abramovich & Brantlinger 1998). Indeed, this single computer application is comprised of many tools that have properties of their physically separate analogues. Sliders (manipulative parameters), electronic tables, random numbers generators, graphical and geometric charts, and other tools available in a spreadsheet environment comprise a non-genetically heterogeneous tool kit. As this paper demonstrates, such a tool kit can bridge two traditionally advanced topics – transformational geometry and matrix algebra – using a variety of tools that reveal a uniform cognitive effectiveness over the whole kit.

In what follows, a spreadsheet-enabled approach to understanding matrices as geometric transformations is suggested. This particular approach is motivated by students' interest in learning about fractals and self-similarity. Peitgen, Jürgens, and Saupe (1992) rely upon the metaphor of a Multiple Reduction Copy Machine (MRCM) and the use of lenses in describing the encoding of fractal images. Here, the focus is upon the matrix representations of lenses and the linear transformations that the lenses, in turn, represent. The authors argue that linear transformations in the Cartesian plane can be easily represented via their effect on a unit square, and, in turn, these resulting lenses are most efficiently represented as matrices. As such, the lenses used in MRCM constitute a fundamental link between two-by-two matrices and linear transformations of the plane. Using a spreadsheet-based tool kit of numerical, analytical, graphical and geometrical notations, one can develop the conceptual links between transformational geometry and matrix algebra that are needed to fully understand both topics.

In order to frame the mathematical ideas of this report, we will first establish the formal relationship between the two topics, and then demonstrate how the tool kit described above can aid students in bridging them conceptually.

## Formal Approach

To formally build the link between matrix algebra and transformational geometry, we can construct a two-by-two matrix to represent each linear transformation of the plane. This construction relies upon the treatment of points in the plane as two-dimensional vectors on which the usual addition and scalar multiplication are defined. First, consider that a line is defined by two distinct points  $(x_b, y_b)$  and  $(x_d, y_d)$  – the first point indicates a base point and the second indicates a direction relative to the base point. Every other point  $(x, y)$  on the line is given by the point  $(x_b, y_b)$  plus some scalar multiple  $t$  of the point  $(x_d, y_d)$ :

$$(x, y) = (x_b, y_b) + t \cdot (x_d, y_d) \quad (1)$$

A linear transformation must take lines to lines. Thus, if the respective images of the points,  $(x_b, y_b)$  and  $(x_d, y_d)$ , under linear transformation  $M$ , are known to be  $M[(x_b, y_b)]$  and  $M[(x_d, y_d)]$ , then

$$M[(x, y)] = M[(x_b, y_b)] + t \cdot M[(x_d, y_d)] \quad (2)$$

For now, let us put one more restriction on  $M$  by insisting that it fixes the origin:

$$M[(0, 0)] = (0, 0)$$

Note that removing this restriction yields an affine transformation - a linear transformation followed by a translation. Thus, throughout this report, we will refer to linear transformations with translations as affine transformations. By using equations (1) and (2), we get the following results:

$$\begin{aligned}
M[(x, 0)] &= M[(0, 0) + x \cdot (1, 0)] = M[(0, 0)] + x \cdot M[(1, 0)] = x \cdot M[(1, 0)] \\
M[(0, y)] &= M[(0, 0) + y \cdot (0, 1)] = M[(0, 0)] + y \cdot M[(0, 1)] = y \cdot M[(0, 1)] \\
M[(x, y)] &= M[(x, 0) + (0, y)] = x \cdot M[(1, 0)] + y \cdot M[(0, 1)]
\end{aligned}$$

Thus, the image of every point  $(x,y)$  in the plane is determined by the images of the points  $(1,0)$  and  $(0,1)$ . To illustrate, let  $M[(1,0)]=(a,c)$ , and let  $M[(0,1)]=(b,d)$ . Then

$$M[(x, y)] = x \cdot (a, c) + y \cdot (b, d) = (ax + by, cx + dy)$$

In other terms,  $M[(x,y)]$  can be defined in a matrix form using four parameters:  $a, b, c$  and  $d$ :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

If we want to allow for translations, we can augment the above matrix by translations  $e$  and  $f$ , in the  $x$  and the  $y$  directions, respectively. Finally, we have the following matrix representation of an affine transformation  $M$ :

$$\begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by + e \\ cx + dy + f \end{bmatrix}$$

We can also represent affine transformation  $M$  geometrically by considering the image of the unit square in the plane. Linearity of  $M$  will force this image to be a parallelogram, which exemplifies the combination of rotations, reflections, dilations, shearings and translations that  $M$  performs on the plane. Figure 1 illustrates an example of an affine transformation that includes a dilation, rotation and translation. The figure also includes labels with the notation mentioned above for matrix representations. While this figure is not part of the tool kit, it indicates a formal relation, through use of notation, between the two representations. An informal (activity-based) approach to this relation is the focus of the first part of the tool kit.

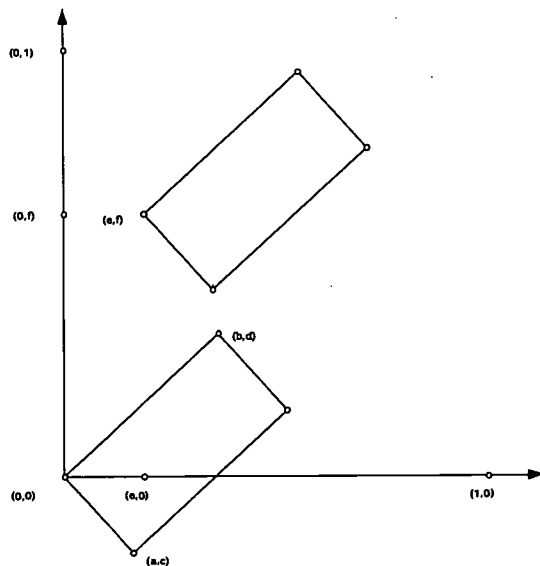


Figure 1: Geometric representation of an affine transformation.

## Activity-Based Approach Using the Tool Kit

As an activity, we suggest that (prospective) high school teachers use the spreadsheet-based tool kit in the following manner. Sheet 1 enables the user to change the six parameters (a, b, c, d, e, f) of the three augmented matrices, each representing an affine transformation of the plane. We can make changes to each real-valued entry in a seemingly continuous manner (in increments of .01) using spreadsheet sliders. Also, the dynamics of such changes is interactively illustrated with spreadsheet graphics, which make it possible to generate a graph of the image of the unit square associated with each matrix (Fig. 2).

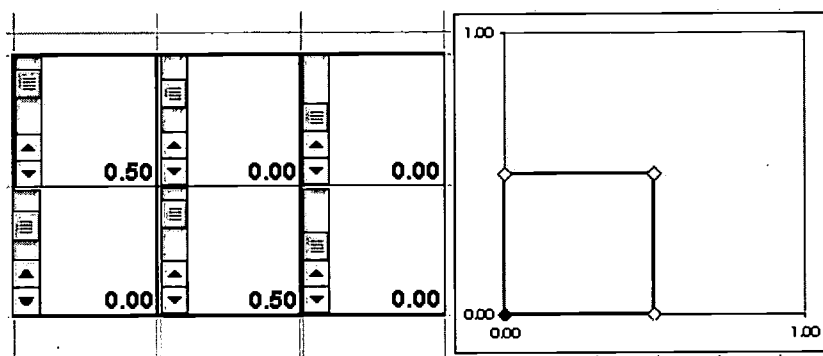


Figure 2: Algebraic and geometric representation of dilation.

The matrix in Figure 2 has entries a, b, c, d, e and f arranged in the conventional manner. We can immediately see the effect of these parameters in the graph of the unit square: the points (1,0) and (0,1) are scaled down to  $(a,b)=(0.5,0)$  and  $(c,d)=(0,0.5)$ , respectively, and the origin is kept fixed at  $(e,f)=(0,0)$ . Thus, the image of the transformation is the unit square dilated about the origin by one half. This image square illustrates that each point in the plane is scaled by one-half, which we can verify by performing multiplication of the matrix M by an arbitrary vector  $(x,y)$ . We can use a similar examination to understand the transformation illustrated in Figure 3, except, this time, the origin is not fixed. Note that, according to Figure 3, the points (1,0) and (0,1) are mapped to  $(a+e,b+f)=(0.5+0.35,0+0.2)=(0.85,0.2)$  and  $(c+e,d+f)=(-0.3+0.35,0.5+0.2)=(0.05,0.7)$ , respectively.

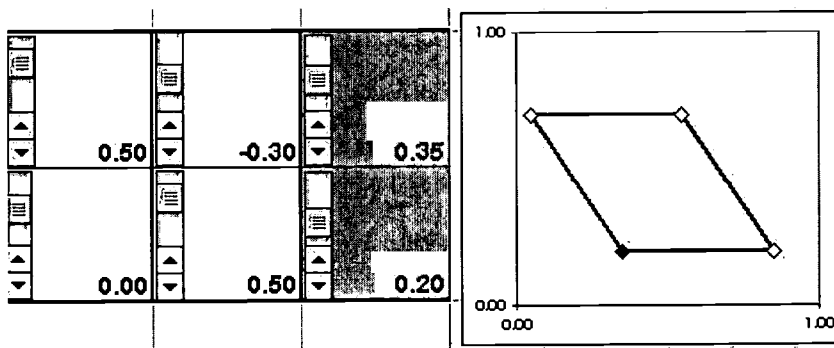


Figure 3: Representations of a combination of dilation, shearing and translation.

Next, students can analyze the values of coordinates of points (recall that these are being treated as two-dimensional vectors) under the transformation of matrix  $M_1$ ,  $M_2$  or  $M_3$  selected randomly at each iteration from the three defined in Sheet 1. The iterations are performed in Sheet 2 by "teaching" the computer to perform matrix multiplication. The formula used to do this is shown in the formula bar of Sheet 2 (Fig. 4). Teachers may find pedagogical value in having students examine this formula. Note, however, that the matrix entries are now listed as rows (rows 4, 5 and 6). The matrix to be used at each iteration is selected randomly as

indicated by the numbers in column C, beginning at C10. By tracking the images of the initial point  $(x_0, y_0)$  after each iteration, we can observe that points are, in general, not revisited. In fact, the pattern as a whole appears very chaotic, as we may expect from a random process. However, the subsequent iteration of a few thousand points yields a nice set, which is due to *the Chaos Game*. The graph of the resulting set is displayed in Sheet 3 (Fig. 5), and it represents a very popular fractal known as Sierpinski's triangle.

A10		=IF(C10=1,A\$4*A9+B\$4*B9+E\$4,IF(C.10=2,A\$5*A9+B\$5*B9+E\$5,A\$6*A9+B\$6*B9+E\$6))									
	A	B	C	D	E	F	G	H	I	J	K
1	X0 ---->		Y0 ---->	0							
2											
3	MATRICES										
4	0.50	0.00	0.00	0.50	0.00	0.00					
5	0.50	-0.30	0.00	0.50	0.35	0.20					
6	0.50	0.00	0.00	0.50	0.25	0.50					
7											
8	x	y	random #								
9	0	0									
10	0.35	0.2	2								
11	0.465	0.3	2								
12	0.4825	0.65	3								
13	0.49125	0.825	3								
14	0.348125	0.6125	2								
15	0.174063	0.30625	1								
16	0.346158	0.353125	2								

Figure 4: The iteration of points by randomly selected matrices.

Sierpinski's triangle is the result of iterating among three matrices, all similar to the one displayed in Figure 2, except  $e_2=0.5$ ,  $f_2=0$  for the second matrix, and  $e_3=0.25$ ,  $f_3=0.5$  for the third matrix. The fractal can be constructed geometrically by repeating the following simple algorithm (Fig. 6): start with a triangle with base and height of length 1 and construct the midpoint segments creating four new triangles; delete the inner triangle and iterate on the outer three triangles. Note that the sketch in figure 6 is used to illustrate the algorithm, but is not part of the tool kit itself. To understand more about the results of the geometric process and our point-wise iteration by randomly selected matrices, we should consider the matrices themselves.

Figure 5: Point-wise generation of Sierpinski's triangle.

Each of the three matrices used for Sierpinski's triangle dilates points about the origin by a factor of one-half. In effect, given a point  $p$  in triangle A (created at an  $n^{\text{th}}$  iteration), matrices  $M_1$ ,  $M_2$  and  $M_3$  without translation map  $p$  to a point in triangle  $B_i$  (created at the  $n+1^{\text{th}}$  iteration). Now, the  $e$  and  $f$  values of  $M_i$  translate this point to  $B_i$ . As this process continues, points get deeper and deeper into the triangles created by the geometric process and ultimately approach points on Sierpinski's triangle.

Arguments like the one described above provide opportunity for insight into the workings of *the Chaos Game*. This phenomenon explains the dense distribution of points in Sierpinski's triangle that we have only begun to demonstrate. A more analytical argument might involve us giving addresses to points (see Peitgen et al) based on the various combinations of randomly chosen matrices. Indeed, the tool kit element displayed in Figure 4 could be modified to aid this analysis as well, but this is left to motivated readers. As further motivation, we can try to generate other interesting fractals by altering the given matrices, thus altering the transformations of iteration.

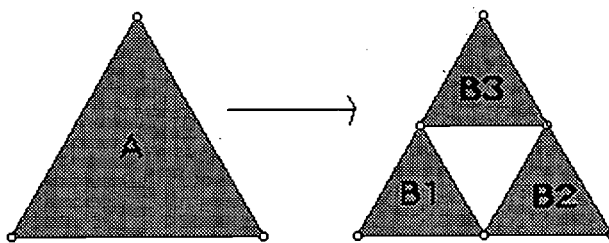


Figure 6: Geometric algorithm for Sierpinski's triangle.

## Closing Remarks

This report illustrates how non-genetically heterogeneous tool kit mediates the unfolding of a fundamental relationship between matrix algebra and transformational geometry. The spreadsheet environment described here provides a meaningful and representational manner in which to explore and discuss these topics jointly, rather than separately. In such a way, it boosts the idea that the appropriate use of technology "blurs some of the artificial separations among topics in algebra, geometry, and data analysis" (NCTM 2000, p. 26). In fact, such artificial separation may lead mathematics educators to question the meaning of matrices outside the context of algebra. On the contrary, we propose a bridge, made possible by appropriate use of spreadsheets that would enhance meanings for both topics. Indeed, we can now view matrices as a useful notation and computation device in describing linear (affine) transformations, and can better see connections between transformational geometry and vectors. Moreover, the computation device can be motivated by and used for the generation of fractals in the plane. It can even be extended to perform in-depth analysis of point addresses in the fractals. The Excel file comprising the tool kit described here can be found under "Papers" at <http://jwilson.coe.uga.edu/EMT668/EMT668.Folders.F97/Norton/Anderson.html>.

## References

- Abramovich, S., & Brantlinger, A. (1998). Tool Kit Approach to Using Spreadsheets in Secondary Mathematics Teacher Education In S. McNeil, J.D. Price, S. Boger-Mehall, B. Robin, J. Willis (Eds) *Technology and Teacher Education Annual*, 1998 (pp. 573-577).Charlottesville, VA: AACE.
- Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. (pp. 515-556). New York: Macmillan.
- National Council of Teachers of Mathematics [NCTM] (2000). *Principles and Standards for School Mathematics*. Reston, VA: (author).
- Peitgen, H., Jürgens, H, & Saupe, D. (1992). *Fractals for the Classroom. Part One. Introduction to Fractals and Chaos*. New York: Springer-Verlag.
- Wertsch, J.V. (1991). *Voices of the mind: a sociocultural approach to mediated action*. Cambridge, MA: Harvard University Press.

# Examining Pedagogical Trends within a Graphing Calculator Environment: An Analysis of Pre-service Teacher Perceptions

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**Abstract:** This paper summarizes the findings of a research study designed to collect pre-service teacher perceptions on factors relating to the use of graphing calculators in the secondary math classroom. The study was designed as a way to help improve instruction in university mathematics methods courses. Data collected as part of the study addresses questions of how to overcome barriers of infusing graphing calculator technology in secondary math instruction, and how to create modeling and problem solving oriented activities that use graphing calculators, but still support a traditional textbook based curriculum. Results from the study suggest that greater emphasis on fostering creativity should be considered in mathematics methods courses. The target audience for this research paper include pre-service teacher educators in mathematics and science.

## Introduction

Despite the current popularity of the new Process and Content Standards from the National Council of Teachers of Mathematics (2000), many teachers of mathematics tend to remain relatively passive in the implementation of new “Standards” based ideas. Although many traditional factors such as time and training constraints contribute to this phenomenon, it appears that the dynamics of classroom instruction emerges even more from a belief system related to the *perceived* demands of the traditional curriculum. New teachers in particular are highly impressionable when it comes to incorporating innovative ideas. Most appear to desire to try new ideas, but in many cases they feel bound to the lecture/seatwork model that is still so pervasive. Experienced teachers of mathematics are even slow to try new ideas because they believe that they must abandon necessary topics in order to include these new ideas as *enrichment*. In addition, instruction tends to be inefficient during the initial attempts at enrichment activities, which adds the threat of more lost time.

The question then becomes how to encourage teachers to incorporate new ideas and methodology while supporting the traditional curriculum. During a short brainstorming session with a group of pre-service math teachers, mathematical modeling and graphing calculator use quickly emerged as potential avenues for enhancing traditional instruction in very meaningful ways. Mathematical modeling and problem solving activities using graphing calculator technology have allowed for traditional curriculum topics to remain the focus while encouraging new ways of thinking about mathematics, as well as providing better contexts and reasons for learning it. However, neither mathematical modeling instruction nor graphing calculator instruction has been largely emphasized in the secondary classroom.

## The Research Study

As a result of these observations, a research study was conducted in which pre-service mathematics teacher perceptions were collected related to the value of infusing graphing calculators into secondary level

mathematics curriculum and instruction. This paper provides a summary of research data gathered from twelve pre-service teachers for the purpose of identifying instructional factors that would lead to conditions in which graphing calculator technology would be used more appropriately and to a much greater extent in the secondary classroom. The research focused on the perceptions of secondary level, pre-service mathematics teachers for several reasons. First, these students had very recently completed collegiate study in mathematics, which does not necessarily emphasize technology based learning environments; consequently, they were likely not to have preconceived notions concerning the use of graphing calculators. Second, all of the students had completed a field placement in secondary school classrooms where they were able to gather the opinions and perceptions of their field mentors. Finally, the views of these pre-service teachers were considered valuable because they were all enrolled in a university mathematics methods course. Information obtained from the research allowed the university to adapt course content and instruction in some very productive ways.

The primary focus of this research was to identify factors that make the calculators and other technology easily adaptable to the classroom, as well as determining the best ways to build mathematical modeling lessons that support existing curriculums. Three research questions were used as guides for the study: 1) what type of graphing calculator technology do pre-service teachers perceive as most conducive to current secondary mathematics curriculum and why, 2) What are pre-service teachers' perceptions of current secondary mathematics curriculum in which graphing calculator technology would be most appropriate for inclusion in the instructional process, and 3) What are pre-service teachers' perceptions of why graphing calculator technology is not commonly incorporated into the instructional process in a mathematical modeling or problem solving format in secondary classrooms.

## Results

Results from the study indicated popular trends for all three questions; moreover, open-ended portions of the questionnaires focused on some very useful suggestions and strategies for how technology might be better infused into the curriculum, even for teachers who prefer a traditional format. Each of the research questions was addressed using an open-ended survey instrument and open dialogue from individual interviews.

Question 1: The TI-83+ emerged as the unanimous choice for most "user-friendly" graphing calculator based on several factors (top three listed): 1) Effective manual descriptions and examples, 2) most logical key sequencing and built-in menus, and 3) only a reasonably short learning time was necessary.

Question 2: The most appropriate course topics that emerged were at the intermediate algebra and pre-calculus levels and encompassed factors related to translations, reflections, and asymptotes for the base functions containing linear, quadratic, absolute value, trigonometric, logarithmic, and exponential components. Additional enrichment topics were mentioned for combining modeling with optimization as a central focus. These most commonly supported existing curriculum but focused on mathematical interpretation.

Question 3: The teachers felt (based on a variety of factors) that the primary reasons their field mentors did not use graphing calculators did not have to do with availability but rather a lack of time to learn the calculator functions. When asked what would influence them to use graphing calculator technology to a greater extent, the following responses emerged: 1) empirical evidence suggesting that higher student test scores would result from implementation, 2) all students in the class having access to the *same model* of calculator, 3) curriculum ready activities designed specifically to support textbook based curriculum.

## Summary

Although these results are in no way exhaustive, they do provide some initial insights into factors that may help graphing calculator technology become more commonplace in the secondary mathematics classroom. Particularly valuable will be the leadership that these teachers provide once they have become established in their own classrooms. In addition, they themselves have gained special insights into the potential value of effective, focused uses of appropriate technology.

# System Development and Fundamental Design of Interactive Mathematical CAI System “MEIKAI”

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**Abstract:** Computers provide opportunities for users including both teachers and students to make knowledge, to gain access to knowledge, and to share knowledge with one another that are not possible before. Our projects consider system environments and minimum requirements for interactive Math CAI systems that must support to assist building students' preparation for the college-level mathematics, and set three basics for Math CAI system such as to keep track of study processes, have the exact interpretation for math symbols, and construct the dynamic curriculum organization. This paper shows brief description of Dynamic Curriculum Control, Algebraic Editor Ma and Easy-To-Use Authoring System.

This work was supported in part by the Ministry of Education, Science and Culture under Grant-in-Aid for Scientific Research No.06452383 and No.08458031.

## Introduction

Private technical colleges and universities have to secure a certain number of students from the management point of view in Japan. Therefore those incoming students' mathematics backgrounds are varied. The Second Subcommittee of the Tokai University Consistency Educational Committee had questionnaires to ask university teachers and students what were in mind for basics and mathematical disciplines from secondary schools to tertiary schools. The analysis focuses on establishment of fundamentals and mathematical disciplines for incoming students. We discussed which mathematical concepts are important for various universities and colleges (Shibata '98). Reforming mathematics curriculum need to be concerned and personal computers provide opportunities for users including both teachers and students to make knowledge, to gain access to knowledge, and to share knowledge with one another that are not possible before. In recent years, awareness of the importance of Mathematical CAI system has increased in the school education. We have observed and tried to resolve the difficulties on Math CAI systems. The establishments of new educational perspectives give guidance to CAI systems' Development (Clements '89). The characteristics are WYSIWYW, WYSIWYG and WYWIWYG. Our interactive *Math CAI* System, MACAI, proposed three basics such as Dynamic Curriculum Control, Algebraic Editor Ma and Easy-To-Use Authoring System.

## Characteristics of MACAI

Our project, the development of MACAI system called “MEIKAI” in Japanese, considers system environments and minimum requirements for interactive math CAI systems that support to assist building of students preparation for college-level mathematics. Our major distinguishing feature among CAI designs is that MACAI should be able to keep track of learning processes, have the exact interpretation of math symbols, and give the individual curriculum organization. The Easy-To-Use authoring system has Text Editor that consists of Algebraic Editor Ma, Simulation Editor, Exercise Editor, Help Editor, and Scientific Language for Geometry, SLanG, based on regular expressions. In the user system, Text Reader that students are able to study based on Dynamic Curriculum Control also has these editors. As the other functions there are preparation windows for students' making and submitting reports under LAN environments. We observe their learning processes and accumulated points by three tables of Students-Points, Time-Points, and Time-Progress. For the authoring and user systems as a new user interface design we introduce MCA, Model and Control Agent, which is enhanced



MVC model from Goldberg (Goldberg '83). Hutchins classifies user interfaces from two axes, Distance-Engagement (Hutchins '86). We introduce Distance-Concurrence as a new user interface classification and furthermore Compatibility-Flexibility evaluation design for CAI contents.

Dynamic Curriculum Control controls the direction of each learner on the CMI Hub Flows that we have established are called the Adaptive Search based on the Genetic Algorithms. The original curriculum DB contains four different levels of contents on the same subject in the MACAI system. The individual curriculum for each student can be chosen from the DB, and set parameters for the allotment of points. The accumulated points determined the course among these contents on Hub Flows. The Algebraic Editor Ma, which provides the consistency of mathematical symbols and formulae, keeps the mathematical meanings of symbols and best human interpretation for the algebraic system Mathematica through the Math Link and the Tex System. In Easy-To-Use Authoring System, SLaNG is the two-dimensional turtle graphics language on the WINDOWS system by using the regular expressions for the simple geometric figures such as regular polygons. The SLaNG, which we have developed, is able to choose from two coordinate systems such as xy-coordinate system and non-coordinate system, change the turtle moving speed, paint the designated territory, and draw regular polygons and ellipses which could not be drawn by conventional graphical applications. This is our purpose to provide a strong support for mathematical CAI systems. Simulation Editor can draw figures in the first year university mathematics, and also numerical solutions of linear systems of equations, numerical integration, evaluation of matrices and determinants, linear regression, Lagrange interpolation, and statistics. Therefore we can change the three tools such as Mathematica, Simulation Editor and SLaNG of drawing figures to suit the occasion. As a result of that, we were able to apply for patents on December in 1996 and as CAI software with the indispensable CMI and the MCA new interface Control Agent Model on October in 1998. Furthermore SLaNG presented on our Internet home page for your convenience. The Handwritten Recognition was also considered especially for mathematical symbols and formulae (Sugamoto & Shibata '98). The research result were released and explained in detail at the Science News in Japan.

## Looking to the Future

To make the interactive electronics book is so expensive. For instance we still need more than several hundreds thousands dollars to complete our project and need to build up a network of supporting groups or organizations. Also the Microsoft upgrade versions were inferior goods such as some upgrade versions did not cover old versions. Therefore every time we have to reconstruct our system programming and pay more money. To get rid of these difficulties we need a new OS and a free language not depending upon Microsoft to construct shell systems and agents for CAI systems. Web version using Java would be the one, and the effective use of programmable calculators could be an alternative.

## References

Clements, M., The Old Industrial Technology and Education, The New Personal Computer Technology and Education, New Media International Symposium on The 50<sup>th</sup> anniversary of Osaka University, pp18-31, 1989.

Goldberg, A., Robson, D., Smalltalk-80 the language and its implementation, Addison Wesley, 1983.

Hutchins, E., Holland, J., and Norman, D. A., Direct manipulation interfaces in D.A. Norman, and S. Draper (Eds) User Centered System Design, New Perspectives on Human-Computer Interaction, Hillsdale, N. J., Lawrence Erlbaum Associates, 1986.

Shibata, M., Fundamental Mathematical Disciplines for Incoming University Students, -from the Report of Tokai University Consistency Educational Committee-, Transactions of Kosen & Daigaku Bukai, Japan Society of Mathematical Education, Vol.5 No.1, pp.147-168, 1998.

Sugamoto, M., Shibata, M., Handwritten Character Recognition by Fuzzy Classification, Rules and Genetic Algorithms, Technical Report of the Institute of Electronics, Information and Communication Engineers, Vol.ET97-105, pp.17-24, 1998.

Yamamoto, H., Shibata, M., Construction of Turtle Graphics Language SLaNG based on Regular Expressions and its application, Technical Report of the IEICE, Vol.ET97-105, pp.25-32, 1998.

# Making Sense of Number: A Resource for Pre-service and In-service Education

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**Abstract:** This paper reports on the development of a multi-media/web/cd package which provides users with opportunities to consider issues in the teaching and learning of numeracy in the early years of elementary school. By viewing exemplifications of classroom activities and listening to commentaries from teachers, children, school managers and parents, users are invited to reflect on and critically evaluate their own practices and contexts. The package also provides a basis for the establishment of an on-line learning community where ideas and resources can be shared and issues discussed.

## Background

The latter half of the 1990s saw a fundamental re-examination of the psychological and philosophical bases of elementary mathematics teaching in the UK. During the previous 25 years the influence of the developmental model of learning, principally associated with Piaget, was all-encompassing. Teachers were encouraged to construe their role as that of facilitator or enabler in contrast to their traditional role as instructor. Mathematics lessons virtually ceased and were replaced by small group and individualised work, often based on the completion of fill-in worksheets, supported by the use of manipulatives (reflecting Piaget's contention that children under the age of 7 are typically incapable of logical thought and therefore require concrete models to make a bridge to the abstract processes inherent in number operations (Piaget 1952). For some years, however, there had been an increasing body of research illustrating young children's possession of many and complex abilities (Donaldson 1978) (Hughes 1986).

The arguments of those who favoured an earlier and more interventionist approach to the teaching of rational counting at the expense of Piagetian pre-number activities such as sorting and matching, (Merrtens 1996) (Thompson 1997) were given credence by the increasing evidence from a number of sources, of declining standards of achievement in basic numerical processes by children in Scotland and England.

In England, a reaction to this has been the establishment of the National Numeracy Strategy which has published a detailed (some would say prescriptive) Framework document (DFEE 1999) which promotes the teaching of mental strategies and advocates a delay in the introduction of standard written methods of calculation. The Framework also provides exhaustive lists of learning targets for students in each year of schooling.

In Scotland, it is recognised that the national curriculum guidelines in mathematics (SOED 1991) are somewhat obsolete in terms of their content and expectations of students. The Scottish Executive, however, sensitive to teachers' oft expressed feelings that they have been subject to a surfeit of innovation, has decided not to review the guidelines immediately and has instead provided only a series of worthy but general suggestions about changing practice. This is the educational vacuum that *Making Sense of Number* seeks to fill.

## Making Sense of Number

The goal of *Making Sense of Number* is to inform and enhance the teaching of numeracy in kindergartens, elementary schools and pre-service teacher education programmes. This action research-based multi-media/web/cd package is designed to illustrate effective learning, teaching and assessment and to provide information about specific activities and resources. It is intended to provide a framework for discussion and development rather than to be used as a manual for teaching.

The national publication, *Improving Mathematics Education 5-14* (SOEID 1997), based on evidence obtained from HM Inspectors reports, the Assessment of Achievement Programme Survey (SOEID 1996) and TIMSS international comparative studies (SOEID 1997), summarised the Scottish situation concerning standards of student achievement and teaching practice in mathematics.

The report focused attention on several areas of concern and recommended the following:

relatively more time to be devoted to teaching number within the overall mathematics curriculum particularly at the early stages

greater emphasis to be given to mental calculation and particularly the teaching of appropriate and flexible strategies

teaching to be more interactive/participative, with fewer groups,

schools to raise expectations for student achievement including, where appropriate, increasing the pace of progression through mathematical topics specified in national curriculum guidelines

schools to further acknowledge the potential role of home learning and to encourage the active participation of parents.

*Making Sense of Number* deals with these issues in a variety of ways, it provides both a structure to help teachers think about and plan the teaching of number and a supporting resource for staff development in this area. It has been designed to enable school and pre-school managers, teachers, student teachers and possibly auxiliary staff and parents to view the types of resources and approaches used in schools actively developing numeracy programmes.

'Making Sense of Number' gives practical help to teachers in identifying useful activities and resources and in assessing learners' progress. By observing the real teachers and children featured in the package, users are encouraged to reflect on their own practice and to relate what they see to their particular classroom or school context. Schools across Scotland were used as sources of the extracts. The package contains some pointers raised from schools who have implemented numeracy programmes over a longer period and who have addressed the implications for later primary years of early intervention (e.g. the impact of an emphasis on the development of mental strategies on the teaching of written methods and the role standard algorithms).

The package also anticipates and facilitates the establishment of an on-line numeracy learning community (via a dedicated website) where 'experts' can be consulted, ideas and resources can be shared, common issues discussed and action research projects and evaluative/qualitative studies developed. It also provides the basis for national training programmes in numeracy as it delivers a consistent message regarding the key principles underlying successful numeracy teaching.

While its main anticipated audience is teachers and student teachers, it could also be used under teacher guidance to inform and train support staff such as nursery nurses and classroom assistants and to help parents understand what schools are seeking to achieve.

## References

- Donaldson, M. (1978). *Children's Minds*. Fontana.
- Hughes, M. (1986). *Children and Number: difficulties in learning mathematics*. Basil Blackwell.
- Merritts, R. (Ed.) (1996). *Teaching Numeracy*. Scholastic.
- Piaget, J. (1952). *The Child's Conception of Number*. Routledge and Kegan Paul.
- Thompson, I. (Ed.) (1997). *Teaching and Learning Early Number*. Open University Press.
- SOED (Scottish Office Education Department). (1991). *National Guidelines: Mathematics 5-14*. HM Stationery Office.

SOEID (Scottish Office Education and Industry Department). (1996). *Assessment of Achievement Programme: Fourth Survey of Mathematics*.

SOEID. (1997). *Third International Mathematics and Science Study (TIMSS): Achievements of Scottish Primary 4 and Primary 5 Pupils*.

SOEID. (1997). *Improving Mathematics Education 5-14*. The Stationery Office.

The National Numeracy Strategy. (1999). *Framework for Teaching Mathematics from Reception to Year 6*. Department for Education and Environment.

# COMPUTER ASSISTED MATHEMATICS LEARNING ENVIRONMENT- A STUDY ON THE COMPUTER, MATH, AND HUMAN INTERACTION

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**Abstract:** Integrating CAI into the math curriculum is a challenging process. Computers change the way teachers teach and the way students learn. The learning process is affected by a combination of factors; for example, learners' attitudes toward math and computers, learners' computer skills, quality and the suitability of the computer learning system, and the way the instructor conducts the course and interacts with the student. This study is designed to explore factors affecting the Guam Community College students' math learning experiences via CAI interactive learning system. The primary instrument for this study was a modified survey questionnaire. In order to gain a fuller insight into the students learning process, the data for this study were also drawn from other sources: (1) student interview; (2) faculty interview; (3) classroom observation; (4) documentation such as student drop-out rate and grades.

## Background of the Study

The island of Guam is the U.S. unincorporated territory in the Western Pacific Rim. It is located between Hawaii and the Philippines. Guam is the largest and most heavily inhabited of the Marianas Islands with a population of 146,000. In addition to the native Chamorro people and "mainland" Americans, Guam hosts many diverse ethnic groups including Filipinos, Chinese, Koreans, Japanese, Pacific islanders from Saipan, Palau, the Marshall Islands, and the Federated States of Micronesia including Pohnpei, Chuuk, Yap, and Kosrae.

With 85% to 90% of college students forced to take remedial math, clearly, students in this region are mathematically challenged. To counter this problem, the Guam Community College introduced interactive multimedia computer-based instructional software into their math courses.

## Perspective

The application of CAI in the area of math has nearly as a long history as CAI itself. The earliest attempt to use CAI to teach math was initiated in mid 1960s. Recent years have seen an increasing use of CAI in mathematics instruction, impacted significantly by rapid advances in computer technology. Research indicates that CAI is most effective when it is targeted to disadvantaged groups such as low achievers and at risk students.

Integrating CAI into the math curriculum is a challenging process. Computers change the way teachers teach and the way students learn. The learning process is affected by a combination of factors; for example, learners' attitudes toward math and computers, learners' computer skills, learners' confidence level in learning math and computers, learners' learning styles, the quality and the suitability of the computer learning system, and the way the instructor conducts the course and interacts with the student. This study is designed to explore factors affecting the Guam students' math learning experiences via a CAI interactive learning system.

## Significance of the Study

This study was inspired by several major concerns related to CAI research.

- (1) Although a substantial CAI research in math is available in the literature, most CAI research was conducted in the 1980s. There is a need to renew CAI research, taking into the consideration that CAI learning systems have made tremendous progress qualitatively in recent years, equipped with sophisticated features not even conceivable a decade ago. "This need is emphasized and reinforced by the increasing use of technological devices in mathematics instruction" (Galbraith & Haines, 2000).
- (2) Major CAI work has been concentrated in the four countries: the United States, the United Kingdom, Canada, and Japan. There is no CAI research conducted in this region, targeting this particular ethnic group. The subjects for this study are Pacific islanders on Guam with the majority of them being local indigenous Chamorro students.
- (3) Most CAI research was conducted in elementary or secondary schools. "There are still many unanswered questions about the effectiveness of CAI for post-secondary students" (Owens & Waxman, 1994; p. 328). This study was conducted on the campus of a community college. As the result, this study will contribute to the literature in the area of post-secondary students' experiences learning math in a computer environment.
- (4) This review of the literature reveals that CAI research in math is limited in utilizing multiple research methodologies. Most research involved one research methodology, either a qualitative method or quantitative method. The present study employs a combination of research methods to gain an insight of the post-secondary students' math learning experiences in a computer environment. The research methodologies in this study include a survey questionnaire, classroom observation, student-researcher e-mail communications, face-to-face interviews with students and the instructor.
- (5) Last, but not the least. The previous studies tended to study learners' attitudes towards CAI as an isolated phenomenon. Factors that impact learners' attitudes towards CAI remain largely unexplored. This study investigates the determinant factors that impact the learners' attitudes towards CAI and introduces a new conceptual framework to explore the complicated interaction of computers, math, and learners.

## The Design of the Study

The primary instrument for this study was a survey questionnaire. A portion of the questionnaire was adapted from the Mathematics-Computing Attitude Scales developed by Peter Galbraith and Christopher Haines (2000).

The survey questionnaire contains nine sections; (1) demographic information (e.g. ethnicity); (2) math confidence (e.g. Math is always a difficult subject for me.); (3) math motivation (e.g. I'd rather spend my time and efforts learning subjects other than math); (4) computer confidence (e.g. I expect to do well if I take a computer course.); (5) computer motivation (e.g. Once I start to work with the computer, I find it difficult to stop.); (6) math learning styles (e.g. When learning new mathematical material I make notes to help me understand and remember.); (7) math-computer interaction (e.g. Visualizations on the computer help demonstrate math concepts.); (8) computer math learning environment (e.g. The computer reduces the necessary interaction between the student and the math instructor.); (9) evaluation of the CAI math learning system (e.g. This software explains and presents math content logically and clearly.). Except for section 1, all the other sections contain 10 items on a Likert scale (1-5).

Mean for each item (except for section 1) will be calculated and the group mean for each section (except for section 1) will be calculated. Factor analysis will be conducted to identify to what extent each factor correlates with math-computer interaction and computer math learning environment.

In order to gain a fuller insight into students learning experiences, the data for this study were also drawn from other sources: (1) student interviews (e-mail interviews and face-to-face interviews); (2) faculty interviews; (3) classroom observation; (4) documentation such as student drop-out rate and grades.

Data collection will span over a period of three semesters. Final report of the study will be completed in the fall of 2001.

## Numeracy CD – Whole Number Concepts and Operations

## Numeracy II CD – Understanding, Using, and Applying Fractions

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**Abstract:** The purpose of this project is to develop a CDROM to address the numeracy problems faced by many adult learners. The CDs will help adults improve their numeracy skills in whole numbers and in fractions. By using multimedia, users will be able to overcome a number of barriers facing adults with low numeracy skills. Most of the learners returning to the classroom feel they simply cannot do math. The traditional style of education did not work for them in the past and by presenting the material in an innovative, interactive environment with audio and visual presentations, we offer these learners a new beginning and added incentive. The programs are interactive and use a step-by-step approach to math, simple language, and specific learning guides.

### Acknowledgments

The development of the Numeracy CD was supported by a grant from the National Literacy Secretariat of Canada, and funds from SIAST Education Equity. The development of the Numeracy II CD was supported by the Department of Post-Secondary Education and Skills Training, Government of Saskatchewan.

### Introduction

Our CDs are being developed in response to requests by educators for material to help teach basic math skills. Literacy levels for many adult learners exceed numeracy levels. Lack of understanding of basic number concepts and the inability to do math computations leave adults in a very difficult situation. Improved numeracy skills are vital to achieving independence and economic goals. Mathematics is a subject that builds on itself and if the basic skills are not there for the foundation, success is difficult to achieve. Our CD programs prepare learners for work-based training opportunities. They are seen as a companion to existing education programs and a bridge to further studies in math.

Learners need a great deal of rehearsal and repetition. CDROM delivery facilitates this well. It directs students who have mastered the concepts to move on quickly and reroutes those having difficulties to a review of the material. The self-instructing and self-paced style of the CDs provides increased flexibility for students and instructors. The programs teach basic math skills, so the content will not change, making it a cost-effective tool.

The scope of this project is broad, both in prospective clients and in areas of use. These programs can be utilized in the areas of literacy, corrections, equity, and remote schooling programs. These CDs are a valuable resource for people with learning disabilities, ESL students, slow learners, youth at risk, remedial learners, and corrections inmates. They can be used by learners in Basic Education, and students in Competency Based Education programs needing to upgrade their math skills in courses such as carpentry, cooking, welding, and mechanics. The universal content and nature of the storylines make the CDs appealing to children and young adults, furthering the field of applicability. The multimedia approach accommodates the diverse needs of learners on campus and in alternate settings, such as home, communities, and the workplace.

## Numeracy CD

### Objectives

- Create an interactive CD program, focused on whole numbers, which uses a step-by-step approach, simple language, and appealing graphics.
- Develop a CD that helps students understand ten basic concepts in whole numbers, concepts vital to understanding and working with numbers.
- Develop a CD that helps students understand how to add, subtract, multiply, and divide whole numbers and apply this knowledge to word problems.
- Provide students and instructors with an additional learning resource that can be easily accessed and allows flexibility with its self-instructing and self-paced style.
- Develop tutorials, drills, practice, tests, and learning aids to achieve the desired outcomes.
- Teach the use of a calculator as part of the process.
- Make learning basic numeracy skills enjoyable.

### Outcomes

- Students will have an understanding of ten concepts in whole numbers.
- Students will be able to apply these concepts to real life situations and be able to use the operations in everyday problem solving.
- Students will be able to use a calculator and will acquire basic computer skills, skills necessary in the majority of employment opportunities.
- Students will have increased their self-esteem through the development of math skills that have eluded them in the past.

### Description of Numeracy CD

The program teaches whole numbers and consists of two parts. The first part explains ten concepts that are important to understanding whole numbers. They are: place values, standard and expanded, rounding, averages, odd and even, greater than - less than - equal to, exponents, prime and composites, order of operations, and Roman numerals.

The second part of the program focuses on the four operations of addition, subtraction, multiplication, and division and includes a unit on word problems to help students learn to choose the correct operation and to solve problems, relating to everyday situations.

The student accompanies George on a journey through the jungle to help save a village facing grave peril. To complete his journey and overcome the obstacles along the way, George must understand whole number concepts. There is a pretest at the beginning of each unit to assess prior knowledge and if understanding of the concept is verified, the student continues to the next unit. If not, the student works through the unit with instruction being provided by a wise parrot named Pinkerton. Numbers, his monkey, provides assistance and comic relief. Practices are included after each idea is covered. If the student demonstrates competence on the practices, he goes to the unit test. If mastery is not evident after a practice or after the test, the student is returned to the instruction section. As George and the student progress through the instruction for each unit, hint charts are established. These charts give an outline of the concept being learned or the steps to follow in order to complete an operation. The student can access each chart by clicking on a toggle switch. A calculator and timestable are built into the program and available to the student by a click of a button. Each unit follows a similar pattern to make the program easy to follow and the instruction is clear and concise. The text on screen is kept to a minimum, with only very important information appearing for clarification. Audio is used extensively to keep the student progressing at a good pace. The program is interesting and entertaining, keeping the attention of the student.



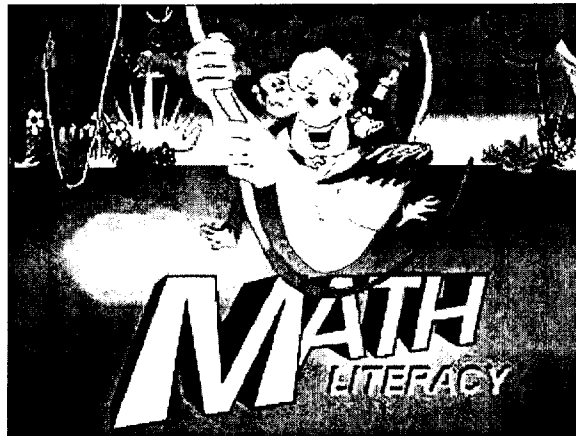


Figure 1: Welcome to Math Literacy

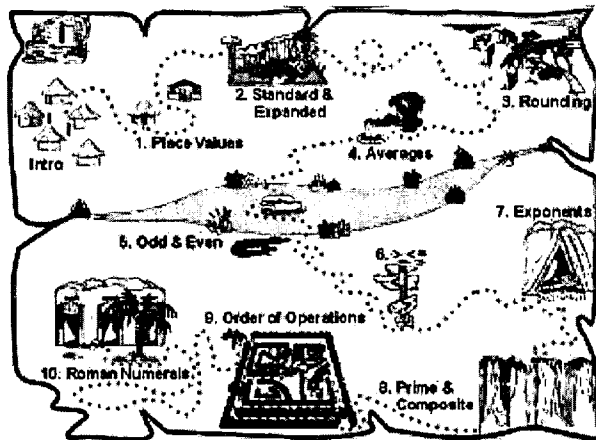


Figure 2: Navigating the Numeracy CD



Figure 3: Instruction Screen from Numeracy CD

## Numeracy II

### Objectives

- Create an interactive teaching CD program, focused on fractions that uses a step-by-step approach, simple language, and appealing graphics.
- Develop a CD that helps students understand fractions and use this knowledge to perform the four operations of addition, subtraction, multiplication, and division and then to apply their knowledge to ratio, proportion, scale, probability, and metric measurement.
- Provide students and instructors with an additional learning resource that can be easily accessed and allows flexibility with its self-instructing and self-paced style.
- Develop tutorials, drills, practice, tests, and learning aids to achieve the desired outcomes.
- Teach the use of a calculator as part of the process.
- Make learning basic numeracy skills enjoyable.

### Outcomes

- Students will have an understanding of fractions and be able to add, subtract, multiply, and divide fractions.
- Students will have an understanding of ratio, proportion, scale, and probability.
- Students will be able to use both standard and metric measurement.
- Students will be able to apply these concepts to real life situations and be able to use the operations in everyday problem solving.
- Students will be able to use a calculator and will acquire basic computer skills, skills necessary in the majority of employment opportunities.
- Students will have increased their self-esteem through the development of math skills that have eluded them in the past.

### Description of Numeracy II CD

The program teaches fractions and has three parts. The first part focuses on understanding fractions. The introduction helps the student to understand fractions, read and write fractions, name the parts of a fraction, and identify the different kinds of fractions. The student learns the concept of equivalent fractions and how to establish equivalency. To manipulate fractions, the student learns how to raise and reduce fractions, compare fractions with like and unlike denominators, and change fractions from one form to another.

The second part of the program then focuses on the four operations of addition, subtraction, multiplication, and division and includes word problems to help the student learn when to use each operation. Each section includes the basic skills necessary to work with fractions in all different forms. The word problems relate directly to everyday practical issues.

The third part of the program utilizes fractions to teach ratio, proportion, scale, probability, and measurement. As part of ratios, the student deals with measurement, time, and money. In the proportion section, the student uses ratio and cross products to find unknowns and to use this knowledge in comparisons. Scale is taught using proportion, and map scale is part of the application. Ratio is used to explain the concept of probability. Measurement includes understanding and using both standard and metric units and applying this knowledge to drawing plans and construction.

In Numeracy II, the learner accompanies a young Aboriginal woman on a journey of discovery through the North where she experiences the beauty of the country, learns about the native plants and animals, and discovers ways to overcome obstacles she faces along the way. In the same way that she learns the ways of nature, she comes to understand fractions, learns how to work with them, and applies her knowledge to solve problems involving fractions. The girl's grandmother accompanies the girl on her journey and provides the instruction.



Figure 1: Journey of Discovery – Numeracy II



Figure 2: Navigating the Numeracy II CD

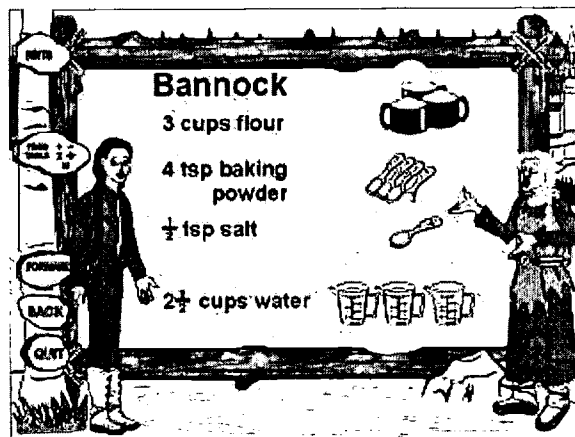


Figure 3: Instruction Screen from Numeracy II CD

## The Reasons

I have instructed in a literacy classroom for ten years. Although the makeup of each class was different, I faced certain similar challenges each year. There was always the challenge of teaching a group of adults from very different backgrounds and with very different skill levels, without holding some back and leaving others behind. Then came the challenge of teaching math to unwilling learners. Most students entering my class felt they simply couldn't do math. Math was a mystery, a subject with a million rules that they didn't have a hope of learning. They hated math and the barriers were up. The reading level in most books was too high, and the problems contained words and ideas that were unfamiliar and irrelevant to the students. The traditional style of teaching had not worked for them in the past. How could I subject them to more of the same?

I needed to find a way to teach math so that it became part of the student's lives, it had relevance, and the stigma was removed. Math skills are essential for everyday living, and most of my students lacked even the basic math skills. I needed to find a way of making math fun for the students. I became an entertainer. I made up stories, I drew pictures, and I created situations that put the students in the problem that needed solving. I tried to show the relevance of math in their lives and the need for an understanding of basic math concepts. The students enjoyed the learning and began to lose their fear of math. They began to make slow, steady progress and their self-esteem grew.

A second teaching method that worked well in class was the use of computers as a teaching tool. Introducing my students to a computer opened a whole new world for them, giving them freedom to explore and confidence in their ability to learn new things. They took great pride in any work they produced and worked diligently to get it done. However, it was difficult to find material at a basic level. Either the programs overwhelmed them with text, or were geared toward children. The most glaring error in most math programs, was the assumption of prior knowledge. Most of the students couldn't work through the programs because they lacked certain skills that the program assumed they had.

## The Results

Armed with a love of math and an understanding of what is required to have students succeed in the basic skills area, I began the task of creating the material for the development of CDs to teach basic math skills to adults. I tried to follow the methods that had worked in the classroom. The CDs entertain. Each CD tells a story, using characters with which the students can associate, and situations that put students in a problem that needs solving. There are wonderful graphics that are a fundamental part of the CDs, adding clarity and visual reinforcement to the instruction. The instruction is clear and concise, using small steps and many examples with relevance to the students. The programs demonstrate patterns for learning and outlines steps needed to work with each concept.

To overcome the reading barrier, text is kept to a minimum and is accompanied by audio. The audio component reduces difficulties created by reading deficiencies, reinforces the instruction, and appeals to auditory learners. The voices interact on the CDs rather than simply reading the text, adding to the students' enjoyment.

The programs have a calculator built into the programs. The student can use the calculator to perform basic math operations by using the mouse to click on the keypad on screen. There is a timestable for those students who cannot memorize math facts, accessed by a click of a button. There are hint charts in each unit, built as the instruction proceeds and accessible for the remainder of the program. These charts allow the students to review steps and procedures at any time. Rather than memorization, the focus is on understanding concepts and operations.

These programs provide one-on-one interaction and allow students to proceed at their own speed. The interactive aspect encourages student participation and gives students a feeling of ownership of their learning. Integrating technology and math instruction, with stories and graphics, can be a powerful way to teach and reinforce basic skills.

## Technology and Basic Math Skills

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**Abstracts:** This paper intends to report on a variety of innovative ways to utilize technology in teaching basic math skills and to motivate students in mastering the basic skills. Teachers will be able to turn so-called boring practice routines into interactive enjoyable learning experience for students to learn the basic concepts necessary for further exploration in the depth of mathematics, therefore to lay a solid foundation. Our experience also showed us that using technology in teaching does not mean simply putting students in front of the computer. The instructional materials have to be carefully designed which will provide opportunities for authentic learning, hence to achieve the outcomes that are expected.

As science and technology have come to influence all aspects of life, mathematics has come to be of vital importance to the educational agenda of our nation. Mathematics is the foundation of science and technology. However, there is a cry for reform in math education because of the poor performance of our students at all levels. It is time for all educators to examine what we are teaching and how we are teaching.

Traditional mathematics teaching is still the norm in our nation's schools. For most students learning mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them. More and more students go into remedial programs that may assist them to obtain their basic math skills. However, the results have not been so impressive. Teachers are facing serious problems and concerned about the poor math skills that may directly impact students' future career and even the quality of their life.

Math anxiety is a widely spread and accepted reality. While many people won't openly admit that they cannot read, they readily admit that they are not good at math. As a result our children have an excuse for poor performance.

Many studies have showed that computer technology has positive impact on learning mathematics among  $k - 12^{\text{th}}$  students. The dynamic and interactive capability of various computer software packages can improve the somewhat static environment that had traditionally existed in classrooms using paper-and-pencil procedures and routines. Technology assisted mathematics activities provide students with an opportunity to explore an even wider range of mathematics problems that ever imagined, and it allows students to visualize the connections among various mathematics topics. Computer technology makes it possible to create a situation in which a graphing utility can be used to enrich the investigative environment for learning mathematics.

The National Council of Teachers of Mathematics (NCTM, 1991) promotes student use of technology to model real-world phenomena that possess a variety of interpretations. Implementation of technology in the classroom can help shift the focus of mathematical ideas from computations and manipulation to modeling and representation of functions or other phenomena in the real world. Mary C. Enderson (1997) in her paper of "Old Problem, New Questions" discusses her experience of teaching students mathematics using computer technology. She believes, "A computer can help eliminate obstacles in doing mathematics – particularly problems involving formulas and calculations." (p32).

More evidence shows that computer assisted instructions (CAI) hold a great deal of promise for providing mathematics instruction to all students, especially to those who suffer from mathematics anxiety which may well be caused by their poor performance in learning mathematics. In addition of offering individualization and flexibility, CAI is impartial, reinforcing, appeals to students' interests, and also offers a sense of privacy for students who are concerned about their performance (Wittman et al. 1998).

Computer technology can positively impact students' learning of basic math skills because it can provide visual effects which can assist learning of better understanding the SEQUENTIAL OPERATIONS

math concepts, therefore they may better apply these math skills learned at school in their real life. Technology used as an effective teaching tool in mathematics education will provide students with opportunities to practice basic skills in an interesting and enriched way, to solve complex and challenging problems, to combine reading, writing skills and communication skills in exploring, discussing and formulating the validity of personally constructed mathematical ideas so that students can draw their own conclusions. Students can use visual aids, drawing, graphics and real-life objects to convince themselves and their peers of the validity of their solutions.

Approximately 104 students in sixth, seventh and eighth grades who have been assigned to participate in the remedial math lab participated in a fourteen-week study. Their selection is based on three criteria: their teacher's recommendation, their last year declining performance and their MAT (Metropolitan Achievement Test) scores (below 21 percentile). These students were not qualified for any special education programs. They were placed in the remedial math lab in an addition to their regular math class to assist them in mastering the basic skills within a semester. Besides attending regular math class everyday, all participants were in the math lab daily for 45 minutes. The students were randomly assigned into one of the two groups. Both groups used a software application on Apple II GS computers for drill and practice to raise the competency level in the areas of addition, subtraction, multiplication and division. The control group used the traditional method, textbooks, paper and pencil exercises and worksheets, while the experimental group used Internet as a practice tool and a multimedia application Astound 2.0 on Macintosh computers. The concepts involved were Surface Area of a Cylinder, basic math facts, subtraction and division problems of 3 or 4 digit numbers.

A Criterion-Referenced test was used to evaluate the growth of students' basic math skills. This set of tests was designed to be used in the district to measure student achievement on the goals set by the district. The evaluation instrument was established by a committee of math teachers within the district, which was comparable to other established instruments to ensure that bias was neutralized and content was sufficient. The test results from this study showed that among the sixth and seventh grade students, the experimental group average criteria-referenced pretest score was 18.75 and the posttest score was 41.63 with an increase of 22.88%. The control group average pretest score was 16.93 and posttest score was 38.22 with an increase of 21.29%. There was some improvement in the MAT test scores in both groups but there was very little difference between the two groups. The average pretest score for the eighth grade experimental group was 37.77 and the posttest average score was 51.05 with an increase of 13.28%, while the pre- and posttest averages in the control group were 40.57 and 45.48 with an increase of 4.91% respectively, which shows significant difference between the experimental group and control group.

It would be difficult to say that there were significant differences in the improvement of basic math skills between the two groups of the 6<sup>th</sup> and 7<sup>th</sup> graders even though the growth in each group was noticeably significant. There was, however, an obvious attitude change among the students in the experimental group according to the teacher's observations. The students enjoyed working on the computers to present the problem-solving strategies and to share their experiences with their classmates. They showed more enthusiasm in learning math skills. The one important lesson we learned as teacher educator and classroom teacher is that simply putting a student in front of a computer does not equate with integrating technology in a math curriculum. Desirable learning outcomes from technology-based learning cannot be realized in the absence of meticulous planning, identification of appropriate resources and materials, and carefully organized presentations dedicated to leading the students through the entire problem-solving process.

#### References:

Enderson, M. C. (1997) Old problems, new questions, *Learning and Leading with Technology*, Vol25, No2, 28-32.

National Council Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, VA: Author

Wittman, T. K.; Marcinkiewicz, H. R. & Hamodey-Douglas, S. (1998). Computer assisted automatization of multiplication facts reduces mathematics anxiety in elementary school children. *Selected Research and Development*, Association for Educational Communications and Technology (AECT), St. Louis, MO

## An On-Line Math Problem Solving Program That Stimulates Mathematical Thinking

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**Abstract.** This paper presents a case study in which the authors developed a Web-based instruction (WBI) unit—an on-line interactive math problem-solving program, and explored the means of using this WBI program to stimulate young children’s mathematical thinking and develop their skills of and interests in solving mathematics problems.

Problem solving is a primary objective for math effective learning, and a required proficiency in real life. Instead of spending a high proportion of instructional time in practicing basic algorithms for the four operations, teachers need to use problem-solving to address computational needs (Peterson et al., 1989); not only teaching how to start from a middle point – to computer with an equation, but also how to proceed from a beginning line – to write an equation using the given data. Problem solving process becomes more and more imperative, especially when technology assists students in computation fluency and accuracy. A calculator does not solve a problem; a human with a calculator does.

Word problems bring various real life situations to a math class, and encourage the students analyze and interpret information as the basis for a solution. Poor reading and comprehension of word problems, and lack of good strategy use are two main causes of the students’ inability in problem solving. (Benko et al., 1999) To lead students to deeper understanding and to avoid depending on cues words, researchers suggest using indirect language (Pawmar, Cawley & Frazita, 1996), i.e. word problems without direct cues words.

A schema is involved when students attempt to understand semantic relationship in a problem (Riley, Greeno, & Heller, 1983). Seven types of word problems are found under the four operations (Vandewalle, 1998). In this program, they are simplified into four types, and further developed into 11 categories.

	Types	Semantic Structures	Semantic Structures
Addition / Subtraction	Join / Separate	Part + Part = Total	Total – Part = Part
			---
	Additive Comparison	Smaller + difference = Bigger	Bigger – Difference = Smaller Bigger – Smaller = Difference
Multiplication / Division	Equal Groups	Group x Each = Total	Total / Group = Each
			Total / Each = Group
	Multiplicative Comparison	Group x Amount = Multiple	Multiple / Group = Amount
			Multiple / Amount = Group

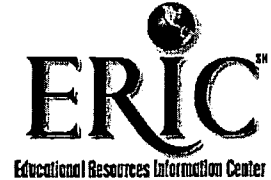
**Table: Semantic Structures of Word Problems**

100 math problems were developed without direct cue words , divided into the four basic operations, and further sorted into 11 categories at three difficulty levels. Studying the semantic structure and recognized characteristics of the known and unknown information helps students understand the relationship among the numbers involved in word problems, and discriminate between operations required to solve word problems.

The KWHL format is used to organize the data. Students analyze the given information—I know, find the goal—I want to find, choose an appropriate operation—how I can find, and write an equation—I have learned. The above process follows the five-step word problem attack strategy, i.e. to recognize the problem, analyze and organize the data, devise a plan, do the computation and check the answer (Howell & Barnhart, 1992). Additionally, using the KWHL format encourages multiple reading of the problems and prolongs mental attention to their semantic structures (Benko et al., 1999). Research shows that US students tended to favor verbal statements over mathematical expression, and less sophisticated methods of solution, e.g. using addition instead of multiplication (Silver et al., 1992). Adopting semantic structures can also promote students’ mathematical expression and encourage them to choose more effective operations.



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